But What do you Mean? -Teacher

Concepts

- Averages and means
- Weighted means

Overview

In this activity, students learn about the concept of mean or average, in addition to learning several ways to find the mean on the TI-Nspire™ handheld (including using a spreadsheet and the mean command). Students also use these methods to find the mean when given the frequencies of each number in a list. Lastly, they discover that the same method can be used to find the weighted mean of a set of numbers. An optional extension has students explore how to calculate harmonic and quadratic means.

Materials

- TI-Nspire™ Math and Science Learning Handheld
- *PTE-ProbStat-WhatDoYouMean_Soln_EN.tns*
- *PTE-ProbStat-WhatDoYouMean_ EN.tns*

Teacher Preparation

This activity is designed to be used in an Algebra 1 classroom, although it can also be used in an advanced Pre-algebra or remedial Algebra 2 classroom.

Students should already be familiar with finding the average of a set of numbers.

- The screenshots within the activity demonstrate expected student results.

- Refer to the screenshots on the final pages for a preview of the student .tns document.

Classroom Management

This activity is intended to be mainly **teacher-led**, with breaks for individual student work. Use the following pages to present the material to the class and encourage discussion.

Students will follow along using their TI-Nspire™ handhelds. The student worksheet helps guide students through the activity and provides a place for them to record their answers.

The .tns document entitled
PTE-ProbStat-WhatDoYouMean_Soln_EN.tns
shows the expected results of working through
the activity.

Information for an optional extension is provided
at the end of this activity, both on the student
worksheet and in the .tns document. If you do
not want students to complete the extension, you
may delete the extension from the .tns document
and have students disregard that portion of the
student worksheet.

Problem 1: The Mean of a Few Numbers

1. On page 1.3, find the average of the numbers
   shown by adding the numbers and dividing
   by the number of numbers (Figure 1).

   - Students who want to complete this
     calculation in one step can use the fraction
     template by pressing ⬡ctrl + ⬡÷.

   Note: If needed, students can press
   ⬡ctrl + ⬡tab to move between work
   areas on one page.

2. Page 1.5 has students use the command
   mean(to find the mean.

   - They can either type the word "mean"
     directly using the alphabet keys or select it
     from the Catalog (⬡📖) (Figure 2).

   Note: The command can also be found by
   pressing ⬡menu ⬡7 ⬡3 ⬡3 for Menu
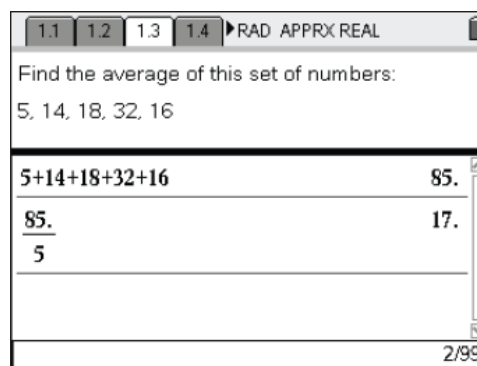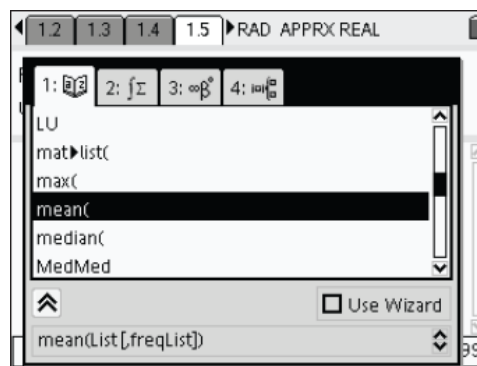   7:Statistics, 3:List Math, 3:Mean.

Figure 1

Figure 2

However, it is just as easy to select it from the Catalog or type it.

3. To use the mean command, enter the numbers, separated by commas, enclosed in braces, within a set of parentheses (which automatically appear if the **mean** command is selected from the Catalog) (Figure 3).

    - The need for both sets of grouping symbols will become more apparent later.

    - On page 1.7, students will find the same set of numbers listed in the first column of the spreadsheet (Figure 4).

4. Find the mean by moving to an empty cell and typing `=mean(a1:a5)`.

    - The colon indicates to use all the numbers from cell A1 through cell A5.

    - When they press ⬡enter, the *calculated* mean will appear in the cell, though the *command* for the mean is still displayed at the bottom of the screen.

The Mean 'Game'

The concept of "mean" is one of the most difficult to verbalize. Most people can tell you how to find the mean, but few can readily say what is the mean *is*! To help students to better understand this idea, they are invited to engage in the "mean game" using the Data & Statistics application.
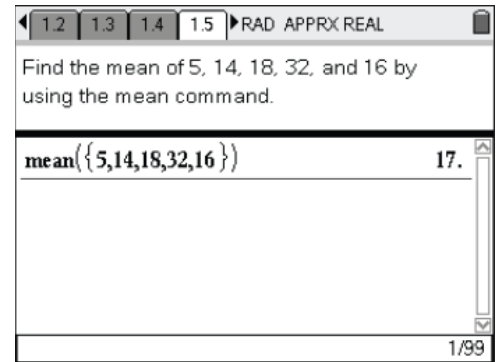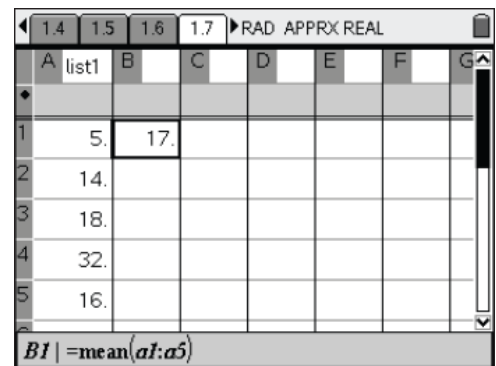
Figure 3

Figure 4

This game simply involves taking one ball from the highest pile and placing it on the lowest pile (Figure 5).

5. "Grab" the top visible ball on highest pile, and drag it to the lowest pile until it reads as close to 1.0 as possible.

6. To "drop" the ball, press ⟨enter⟩, move the cursor off of the pile, and press ⟨⟩ .

This process should be continued until all piles have the same height—the **mean** of the original set of scores!

In this example, the scores are those that students have already been working with, and the average should be 17.

This activity should help students to better appreciate that the mean represents the "balance point" for a set of scores—the value around which the scores are evenly distributed. Students may be challenged to put this idea into their own words, before continuing with the remaining activities.

7. Have students complete pages 1.11 and 1.12 independently to practice each of the methods (Figure 6).

8. They will be able to verify if they completed each method correctly by checking that all three give the same result.
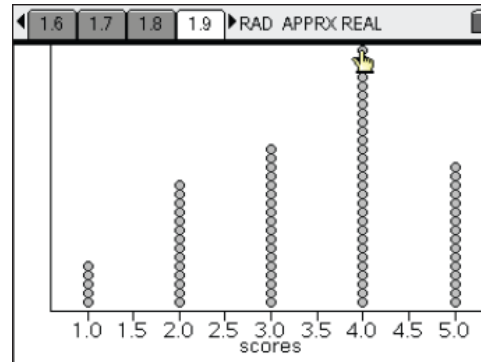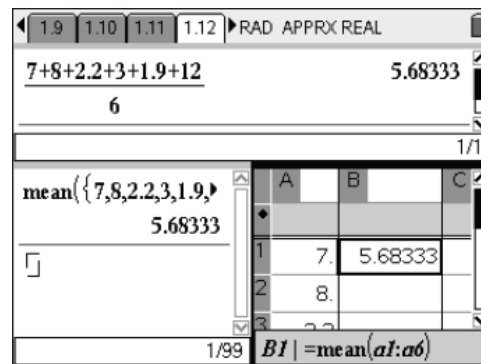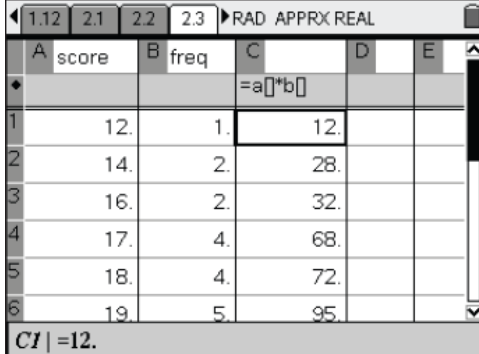
Figure 5

Figure 6

Problem 2: The Mean of Many Numbers

9. Discuss the material on pages 2.1 and 2.2, and ask how the mean might be found without having to list and sum all 20 numbers.

   - On page 2.3, the scores earned are listed once each in Column A.

10. Enter the corresponding frequencies into Column B.

11. Display the corresponding products in Column C by entering =a[ ]*b[ ] into the gray formula cell marked with a diamond symbol (See Figure 7).

   - The [empty] open and closed brackets indicate to use corresponding cells, so that the value in A1 is multiplied by the value in B1, with the product displayed in C1, and so on.
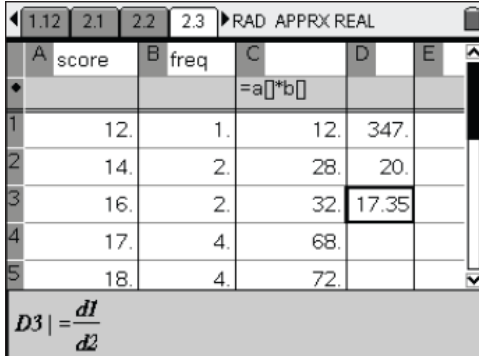

Figure 7

The formula to find a sum in a spreadsheet is similar to the formula to find the mean.

   - For example, the sum of the frequencies can be found by typing =sum(b1:b7).

12. Find the sums of the values in Columns B and C and then divide to find the mean (in cells D1, D2, and D3, respectively; see Figure 8).

13. Advance to the Calculator application on page 2.4.

Figure 8

14. To use the mean command when given a list of numbers and the frequency of each number, list the numbers in one set of braces and the corresponding frequencies in a second set of braces.

15. Enclose both sets in parentheses, separated by a comma (See Figure 9).

   • Stress that the frequencies must be listed *in the same order* as the corresponding values.

16. Have students complete pages 2.5 and 2.6 independently to practice each method (Figure 10).

17. They will be able to verify if they completed each method correctly by checking that they give the same result.

Problem 3 – The Weighted Mean

18. Read page 3.1, and ask for students' suggestions to finding the mean.

19. Then direct their attention to page 3.2.

20. When ready, have them use the Lists & Spreadsheet and Calculator applications on page 3.3 to find the mean (Figure 11).

21. On page 3.4, explain that the mean they found is called the *weighted mean* because different values have different weights of importance.

   • The test is more important than the quizzes, so it makes up a greater percentage of the mean.
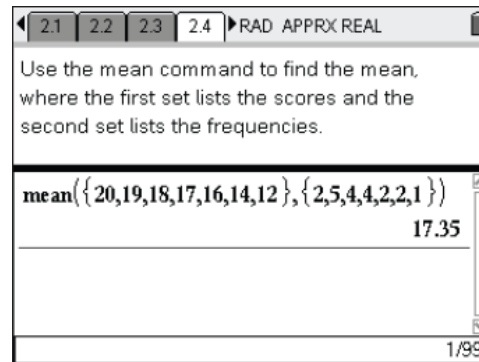
Figure 9
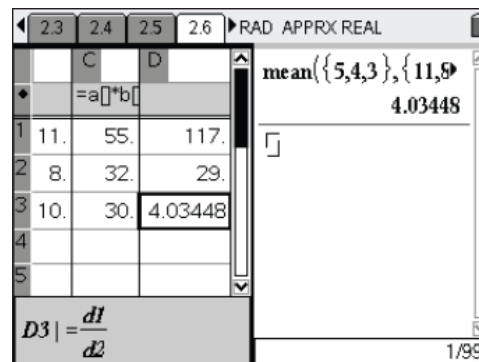
Figure 10
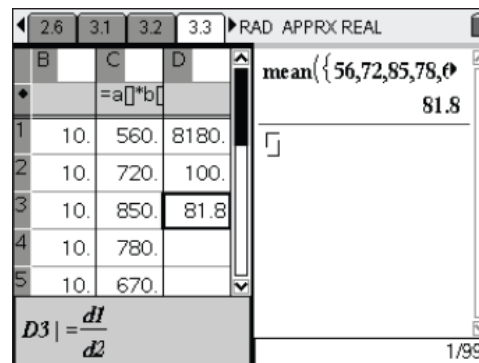
Figure 11

22. Find the mean of the six scores as if the scores were not weighted (Figure 12).

23. Discuss why the weighted mean for this example is greater than the "regular" mean.

Figure 12

24. Have students complete pages 3.5 and 3.6 independently to practice each method (Figure 13).

25. They will be able to verify if they completed each method correctly by checking that they give the same result.

26. If time permits, ask students to predict what the mean would have been if the scores were not weighted.

Figure 13

27. Then have them find this score (85.75).

28. Discuss why it is higher than the weighted mean.

Extension

29. Have students find the harmonic mean and quadratic mean of the given set of numbers (Figures 14 and 15).

30. They may also research some other means and their applications.

Figure 14

Figure 15

But What do you Mean?

(Student)TI-Nspire™ Document: *StatAct02_WhatDoYouMean_EN.tns*

<table>
<tr>
<td>

1.1 1.2 1.3 1.4 ▶ RAD APPRX REAL

_____

BUT WHAT DO YOU MEAN?

_____

Statistics

Calculating means
and weighted means

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1.1 1.2 1.3 1.4 ▶ RAD APPRX REAL

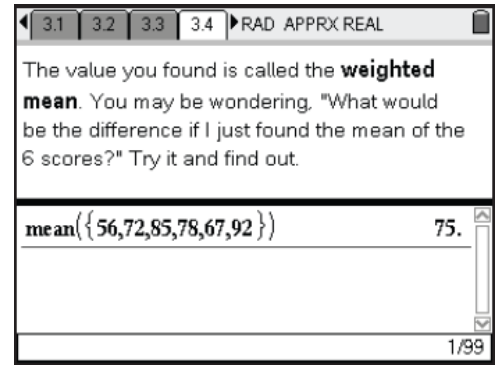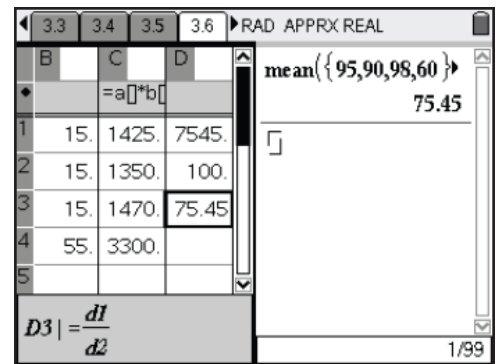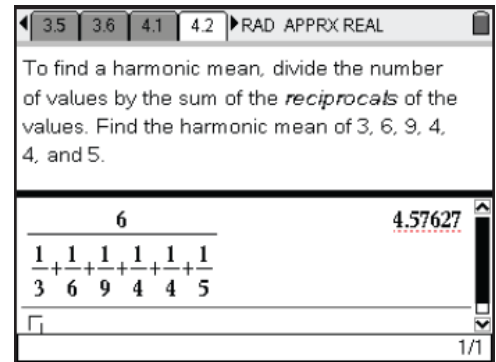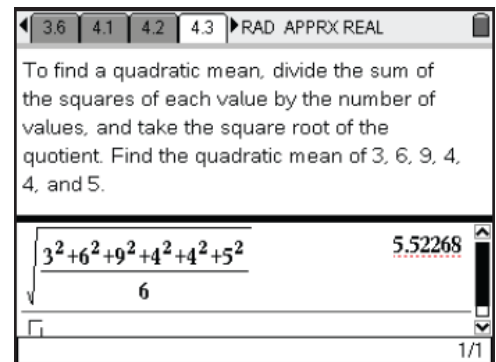It is easy to find the **average** of a group of
numbers: just add all the numbers together
and divide the sum by the number of numbers.

In this activity, we will look more closely at this
idea of average and what it means!

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1.1 1.2 1.3 1.4 ▶ RAD APPRX REAL

Find the average of this set of numbers:

5, 14, 18, 32, 16

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1.1 1.2 1.3 1.4 ▶ RAD APPRX REAL

The average of a set of numbers is also called
the **mean**.

On the TI–Nspire handheld, you can find the
mean of a set of numbers by using the mean
command.

mean(

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<td>

1.2 1.3 1.4 1.5 ▶ RAD APPRX REAL

Find the mean of 5, 14, 18, 32, and 16 by
using the mean command.

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1.3 1.4 1.5 1.6 ▶ RAD APPRX REAL

A spreadsheet can also be used to find the
mean.

Use the mean command in the spreadsheet
on the next page to find the mean of the
numbers in cells A1 through A5.

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1.5 1.6 1.7 1.8 ▶ RAD APPRX REAL

Now we know how to find the mean (or
average) of a set of numbers—but what are
we actually doing?

What does the mean *mean*?

On the next page, play the **mean game** by
dragging points one at a time from the highest
pile to the lowest, until all the piles are the
same height.

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1.4 1.5 1.6 1.7 ▶ RAD APPRX REAL

A list1	B	C	D	E	F	G
1 5.						
2 14.						
3 18.						
4 32.						
5 16.						

A1 |

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1.6 1.7 1.8 1.9 ▶ RAD APPRX REAL

1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0
scores

</td>
</tr>
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1.7 1.8 1.9 1.10 ▶ RAD APPRX REAL

Question

Describing the mean in words (NOT just how
to find it!) is a very hard thing to do. Can you
define the mean of a set of numbers?

Answer ≫

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1.8 1.9 1.10 1.11 ▶ RAD APPRX REAL

To review, use the applications on page 1.12
to find the mean of the numbers below, using
each of the three tools provided.

7, 8, 2.2, 3, 1.9, 12

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<td>

1.9 1.10 1.11 1.12 ▶ RAD APPRX REAL

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	A	B	C
1			
2			
3			

0/99 **A1** |

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</table>
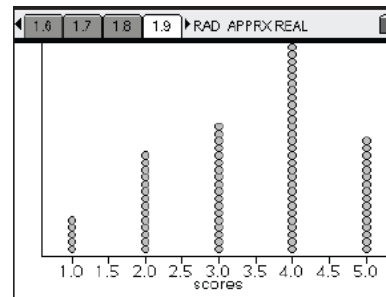
Screen 1 (1.10 | 1.11 | 1.12 | 2.1 | RAD APPRX REAL)

Suppose that a teacher gave a class of 20 students a quiz, worth 20 points. Two students got a perfect 20, 5 got a 19, 4 got an 18, 4 got a 17, 2 got a 16, 2 got a 14, and 1 got a 12 (but won't admit it).

Is there a better way to find the mean score than to list and sum all 20 scores? How would you do it?

Screen 2 (1.11 | 1.12 | 2.1 | 2.2 | RAD APPRX REAL)

On the next page, each score is listed in Column A. List the frequency (number of people) for each score in Column B.

Find the mean of the scores by multiplying each score by its frequency, and then dividing that sum by the sum of the frequencies.

Screen 3 (1.12 | 2.1 | 2.2 | 2.3 | RAD APPRX REAL)

A score	B freq	C	D	E
12				
14.				
16.				
17.				
18.				

A1

Screen 4 (2.1 | 2.2 | 2.3 | 2.4 | RAD APPRX REAL)

Use the mean command to find the mean, where the first set lists the scores and the second set lists the frequencies.

0/99

Screen 5 (2.2 | 2.3 | 2.4 | 2.5 | RAD APPRX REAL)

To review, use the next page to find the mean of the quiz scores, using both of the ways shown.

11 students got a 5

8 students got a 4

10 students got a 3

Screen 6 (2.3 | 2.4 | 2.5 | 2.6 | RAD APPRX REAL)

A	B	C

A1   0/99

Screen 7 (2.4 | 2.5 | 2.6 | 3.1 | RAD APPRX REAL)

Now suppose 5 quizzes are each worth 10% of your final class grade and a test is worth the remaining 50%.

How can you find your final class grade if your quiz scores are 56, 72, 85, 78, and 67, and your test score is 92?

Screen 8 (2.5 | 2.6 | 3.1 | 3.2 | RAD APPRX REAL)

If you said similar to the way you worked in Problem 2, then you are correct.

The situation is the same as if you took 100 tests, and received a score of 56 on 10 of them, 72 on 10 of them, and so on. Use the next page to try the calculations.

Screen 9 (2.6 | 3.1 | 3.2 | 3.3 | RAD APPRX REAL)

A	B	C

A1   0/99

Screen 10 (3.1 | 3.2 | 3.3 | 3.4 | RAD APPRX REAL)

The value you found is called the **weighted mean**. You may be wondering, "What would be the difference if I just found the mean of the 6 scores?" Try it and find out.

0/99

Screen 11 (3.2 | 3.3 | 3.4 | 3.5 | RAD APPRX REAL)

To review, suppose your teacher gave you 4 tests 3 worth 15% each and the last worth 55%. You received 95, 90, and 98 on each of the first three, and, who knows how, got a 60 on the last test. On the next page, find the weighted mean both ways.

Screen 12 (3.3 | 3.4 | 3.5 | 3.6 | RAD APPRX REAL)

A	B	C

A1   0/99

| ◄ 3.4 | 3.5 | 3.6 | 4.1 | ►RAD APPRX REAL | 🔋 |

Extension

Did you know that there is more than one "mean" in mathematics? We have been working with the **arithmetic mean**.

Two different types of means, used in certain situations such as working with speed and voltage, are the **harmonic mean** and the **quadratic mean**.

| ◄ 3.5 | 3.6 | 4.1 | 4.2 | ►RAD APPRX REAL | 🔋 |

To find a harmonic mean, divide the number of values by the sum of the *reciprocals* of the values. Find the harmonic mean of 3, 6, 9, 4, 4, and 5.

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| ◄ 3.6 | 4.1 | 4.2 | 4.3 | ►RAD APPRX REAL | 🔋 |

To find a quadratic mean, divide the sum of the squares of each value by the number of values, and take the square root of the quotient. Find the quadratic mean of 3, 6, 9, 4, 4, and 5.

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| ◄ 4.1 | 4.2 | 4.3 | 4.4 | RAD APPRX REAL | 🔋 |

Research

Can you find some applications of these means?

Can you find other means and where they are used?