How Fast is the U.S. Population Growing? -Teacher

Concepts

- Data representation and interpretation
- Visual transformations of exponential functions
- Exponential regressions

Materials

- TI-Nspire™ Math and Science Learning Handheld
- *PTE-NumOps_USPopulation_EN.tns*
- *PTE-NumOps_USPopulation_Soln_EN.tns*

Overview

Students are introduced to modeling exponential data through an investigation of the population of the United States. Students use multiple representations to explore aspects of the growing population. Data was collected from the U.S. Census Bureau for the dates 1860 to 2006.

Teacher Preparation

This investigation offers opportunities for review and consolidation of key concepts related to exponential, quadratic, cubic, and logarithmic functions. As such, care should be taken to provide ample time for ALL students to engage actively with the requirements of the task, allowing some who may have missed aspects of earlier work the opportunity to build a new and deeper understanding.

At the Algebra 2/Precalculus level, this activity can serve to consolidate earlier work on exponential functions. It offers a suitable introduction to exploring exponential data, model fitting using exponential functions, and interpretation of graphs.

Begin by reviewing with students the general exponential form of $y = ab^x$.

- The screenshots contained with the activity directions demonstrate expected student results.
- Refer to the screenshots at the end of this document for a preview of the student .tns document.
- Open *PTE-NumOps_USPopulation_EN.tns* on the TI-Nspire™ handheld.

Classroom Management

This activity is intended to be teacher-led with students in small groups. You should seat your students in pairs so they can work cooperatively on their TI-Nspire™ handhelds.

You may use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds, although the majority of the ideas and concepts are only presented in this activity document. Be sure to cover all the material necessary for students' total comprehension.

The student worksheet is intended as an investigation through the main ideas of the activity. It also serves as a place for students to record their answers. Alternatively, you may wish to have the class record their answers on a separate sheet of paper or just use the questions posed to engage a class discussion.

Suggestions for optional extension questions are provided at the end of this activity.

This investigation explores the growth rate of the United States population from 1860 through 2006. The years have been modified so that "0" represents the year 1800.

Throughout the activity, students will develop exponential models to make conjectures based on interpolation and extrapolation of the data.

1. Students should first study the data in the spreadsheet on page 1.3 and record any observations on their worksheets (Figure 1).
2. Encourage them to discuss their impressions and observations of the data. You may want to pose questions about what type of function would best model the data.
   - On page 1.4 is a Graphs & Geometry application with an appropriate window for this data and pre-constructed sliders at the top of the page.
3. Create a scatter plot of the data by pressing (menu) (3) (4) for Menu 3:Graph Type, 4:Scatter Plot.
   - The function entry line appears.
4. Press (enter) to open the drop-down menu for the *x*-variable, and select `year`.
5. Press (tab) (enter) to move to the drop-down menu for the *y*-variable, and select `uspop` (Figure 2).

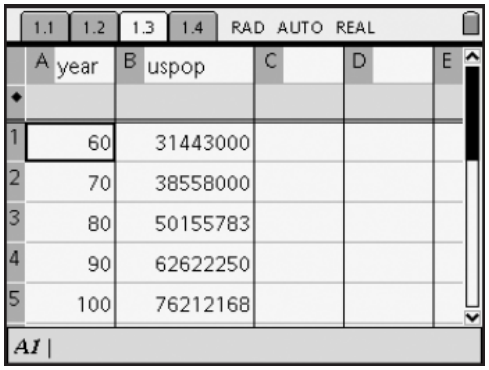

Figure 1

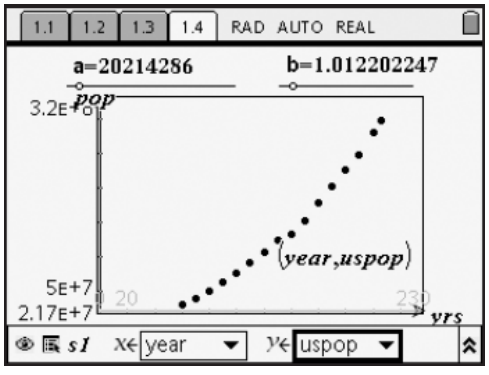

Figure 2

6. Select the Point On tool from the Points & Lines menu, and click on a data point—this allows tracing of the data (Figure 3).
7. Press enter.
8. With the point selected, press menu 1 3 for Menu 1:Actions, 3:Attributes to change the appearance of that point to be an open circle (to increase its visibility).
9. Press enter and then either the right or left direction keys to choose the desired appearance: circle, empty circle, square, empty square, cross, plus, thin.
10. Press enter.
    - If the *y*-coordinate appears in scientific notation, instruct the students to hover the cursor over it and press the ctrl key to display the number in standard notation.
11. Press ctrl + G to unhide the function entry line.
12. Next, press menu 3 1 for Menu 3:Graph Type, 1:Function to change graphing modes, key in "$a \cdot b^x$" for **f1**, and press enter (Figure 4).

Note: It is important that students enter the multiplication symbol between the pre-defined variables *a* and *b*. Otherwise, the device searches for the variable named "*ab*."

The graph of the function and the function rule now appear on the screen.

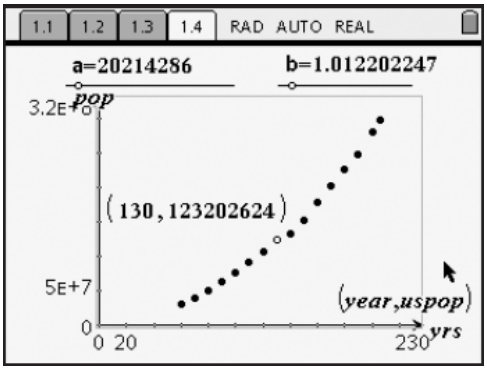

Figure 3

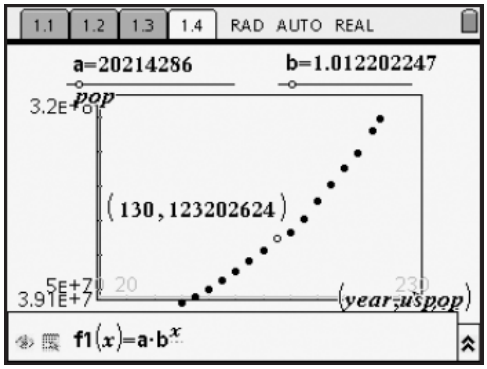

Figure 4

13. Press ctrl + G once again to hide the function entry line (Figure 5).
    - Students may also grab and drag text boxes and rearrange the screen so as to provide more organization and better visibility.
14. Grab and drag the points on the **a** and **b** sliders until they have a visually "good fit" of the data (Figure 6).
    - One sample equation is given by: **f1**(x) = 20,026,786 · 1.013280899^x
15. This would be a good opportunity to discuss why they choose the values they did to get the fit they have.
16. Once more use the Point On tool—this time to place a point on the function (Figure 7).
17. Again, students may wish to use the Attributes tool to change the point's appearance and press the ctrl . key while hovering over the point's coordinates to adjust the display digits.
18. Next, insert a new page by pressing ctrl + I and selecting 1:Add Calculator.
19. Perform an exponential regression by pressing menu 6 1 A for Menu 6:Statistics, 1:Stat Calculations, A:Exponential Regression.
20. To set up the regression, select `year` as the X-List and `uspop` as the Y-List.

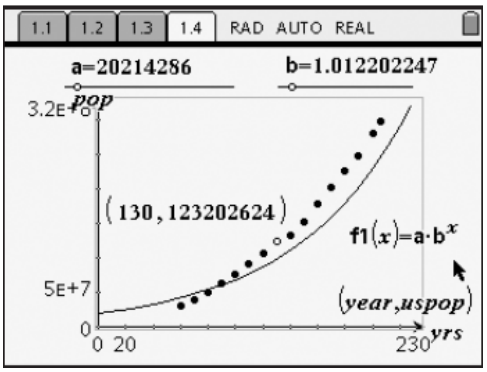

Figure 5

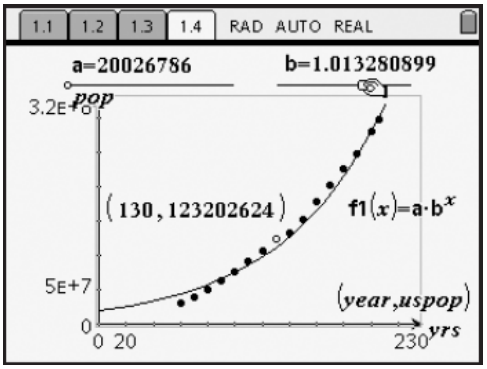

Figure 6

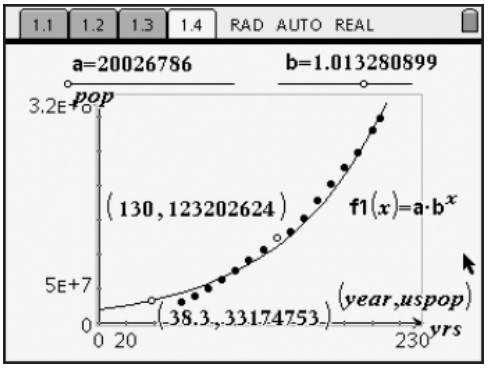

Figure 7

- Press (tab) to move between the drop-down menus.

21. Make sure the regression equation is set to be saved as `f2`, and press (enter) to compute the regression (Figure 8).

22. Now return to page 1.4, unhide the function entry line ((ctrl) + (G)), and press ▲ on the NavPad cursor controls to access `f2` if needed.

23. Press (enter) to display its graph, and change its Attributes so the graph appears as a dashed line (Figure 9).

24. Repeat placing a point on this function, thus enabling it to be traced, altering its attributes and coordinates as desired.

Students now have the information they need to explore the U.S. population!

Have them complete the exercises on the student worksheet, or use those questions to foster a whole-class discussion.

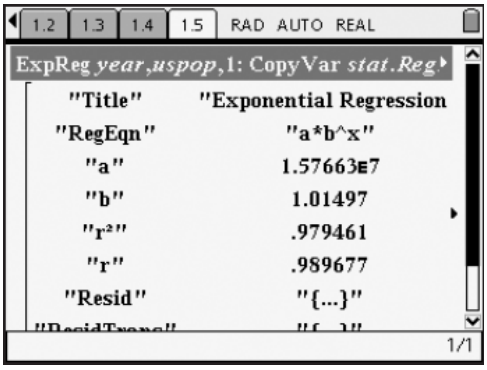

Figure 8

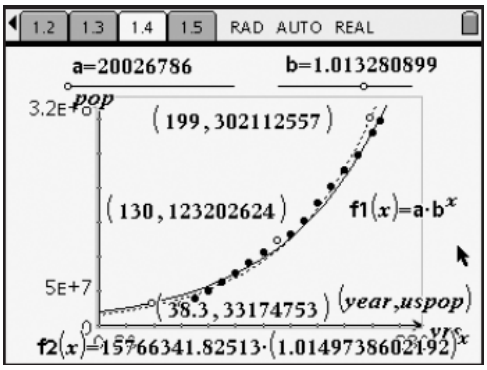

Figure 9

Solutions – Student Worksheet Exercises

Answers will vary. Accept reasonable explanations.

1. ≈1.5%. Possible answer: In exponential equations of the form $y = a(1 + b)^x$, b represents the percent of increase. In the regression equation, $1 + b \approx 1.015$, so $b \approx 0.015$, or 1.5%.
2. a = population of the U.S. in 1800; answers will vary. Accept reasonable explanations.
3. Population in $2020 \rightarrow x = 220$; $f2(220) \approx 414{,}762{,}602$; accept reasonable predictions for f1.
4. Population in $1776 \rightarrow x = -24$; $f21(-24) \approx 11{,}036{,}068$; accept reasonable estimates for f1.

Optional Extension Questions

1. Use residuals and Σ^2 to compare the two models.
2. Use other types of functions to model the data.
3. Gather data for the populations of India or China and compare initial values and growth rates with that of the U.S.
4. Compare population versus land mass.

How Fast is the U.S. Population Growing?

(Student)TI-Nspire™ Document: *Alg2Act1_USPopulation_EN.tns*