The Golden Ratio-Teacher

Concepts

- Ratios
- Square roots and irrational numbers
- Regular polygons

Materials

- TI-Nspire™ Math and Science Learning Handheld
- *PTE-NumOps_GoldenRatio_EN.tns*
- *PTE-NumOps_GoldenRatio_Soln_EN.tns*

Overview

Students begin this activity by exploring two number patterns that approach the value of the Golden Ratio, which is approximately 1.618. Then they work with a partner to find the ratio of their height to the distance from their head to their fingertips. They discover that this ratio, and especially the class mean of this ratio, is also amazingly close to 1.618. Lastly, students construct figures with the dimensions that form the Golden Ratio and learn the exact value of the Golden Ratio, $\dfrac{1+\sqrt{5}}{2}$

Teacher Preparation

This activity was originally designed to be used in a high school geometry classroom, but it is also appropriate for a lesson on numbers and operations. This activity allows students to observe the Golden Ratio appear as the result of several types of activities before introducing its exact value.

This lesson is best done with students working in pairs, especially for Problem 2 which requires measuring. Provide cloth or steel tape measures for each pair. If tape measures are not available, students can use yardsticks or meter sticks.

The screenshots within the activity demonstrate expected student results. Refer to the screenshots at the end of this activity to see the original student .tns document.

Classroom Management

This activity is intended to be primarily teacher-led, with breaks for independent student practice.

Use the following pages in conjunction with the student worksheet to present the material to the class and encourage discussion; students will follow along using their handhelds. The majority of the ideas and concepts (including the instructions for the Graphs & Geometry constructions in Problem 3) are only presented in this document, so be sure to cover all material necessary for students' total comprehension.

When students are working independently, be sure to walk around the room and help as needed.

The student worksheet is intended to guide students through the main ideas of the activity and serve as a place for students to record their answers.

The .tns document entitled *PTE_NumOps_GoldenRatio_Soln_EN.tns* shows the expected results of working through the activity (calculations are evaluated and Graphs & Geometry pages are constructed).

Information for an optional extension is provided at the end of this activity. It is also provided on the student worksheet and in Problem 5 of the student .tns document.

Problem 1 – Number Patterns

1. Ask students to read the description of the pattern on page 1.2 and then move to the Calculator application on page 1.3 to try it (Figure 1).

Figure 1

2. Remind them that only non-negative numbers have real square roots, so they must choose a non-negative number to begin.

3. Students should repeat the process of taking the square root and adding 1 at least ten times.

   - They can press ⬡ctrl ⬡ans(–) to automatically bring down the previous answer instead of entering it, or they can simply type ans using the green alphabet keys.

Figure 2

4. Have students share their final results.

   - All results should be near 1.618 (Figure 2).

   - If any final numbers are not close to that, have them continue the process of taking the root and adding 1 a few more times.

   - They should find it interesting that regardless of the initial number, the final results are approximately the same.

5. To explore another pattern, present the first few terms of the Fibonacci sequence:
   1, 1, 2, 3, 5, 8, 13, 21, 34, 55 …

6. Ask students how it is formed.
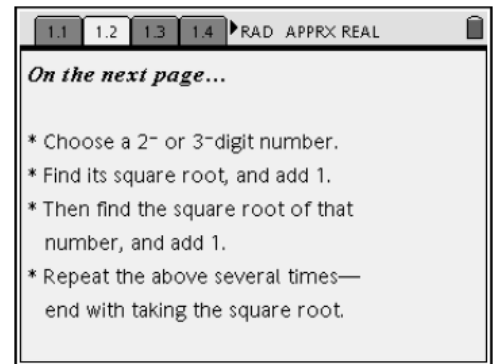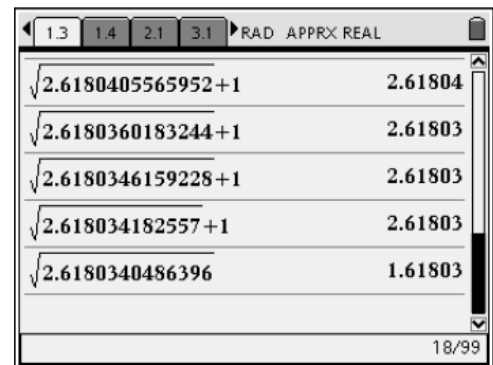
   - *Each term is the sum of the previous two terms.*

7. Have them then investigate what happens as each term of the sequence is *divided* by the term before it (1 ÷ 1, 2 ÷ 1, 3 ÷ 2, 5 ÷ 3, …).

8. Students should find that these ratios also get closer and closer to 1.618 (Figure 3).

   • They might need to press (ctrl) (tab) to move between the Notes and Calculator application work areas.

Figure 3

Problem 2 – A Human Body Ratio

In this problem, students will work with a partner to find the ratio of their height to the distance from the top of their head to the end of their fingers (when their arms are down against their sides). It does not matter if students choose to measure in inches or centimeters—as long as they use the same unit for both measurement—but encourage them to measure as precisely as possible.

9. As they find their ratios, have students write them on the board.

10. Then show them how to find the *mean* of their ratios, by press (menu) (7) (3) (3) for Menu 7:Statistics, 3:List Math, 3:Mean.

11. Although parentheses appear when selecting the Mean tool, students should enclose the list of ratios inside braces by pressing (ctrl) ({}) (Figure 4).

   • Results will vary, but if students measured carefully, the mean of all of the ratios should fall around 1.618.

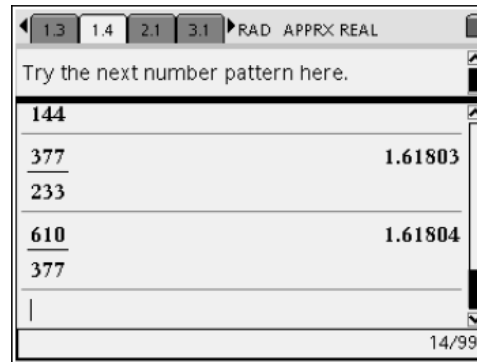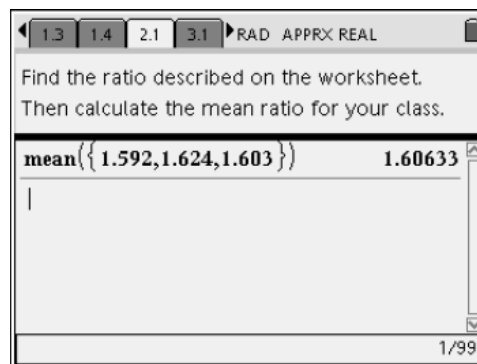**Figure 4**

There are more body measurements whose ratios are approximately 1.618. Students can investigate another human body ratio as part of an extension to this lesson (see Problem 5).

Problem 3 – Some Special Constructions

12. On the square shown on page 3.1, use the Midpoint tool from the Construction menu to locate the midpoint of the base (Figure 5).

13. Then construct a Segment that joins this midpoint to the upper-left vertex of the square (⬡menu ⬡6 ⬡5 for Menu 6:Points & Lines, 5:Segment) (See Figure 5).

Figure 5

Now construct a circle in such a way that the segment just drawn is a radius of the circle.

14. To do so, use the Compass tool from the Constructions menu, and click the segment to indicate that this is the length to use for the radius (Figure 6).

15. Click the midpoint of the base of the square, identifying it as the center of the circle.

Figure 6

16. Extend the top and bottom sides of the square so that they extend past the "left-most" point of the circle.

17. This can be done by using the Line tool (from the Points & Lines menu) to overlay a line on each side and then extending the lines as needed.
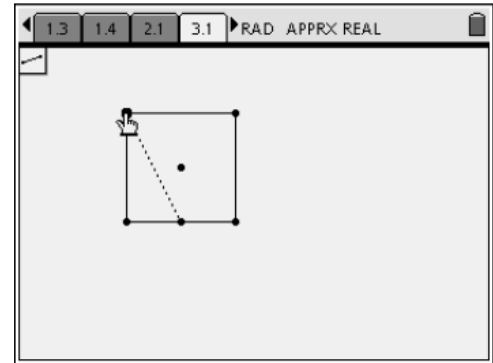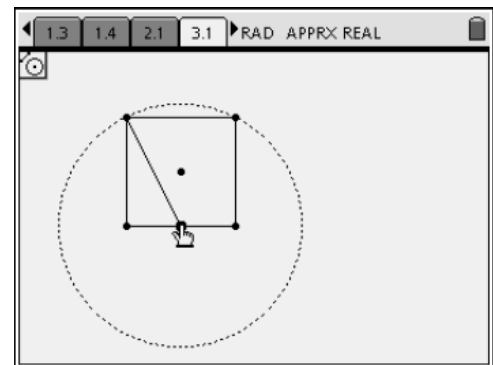
18. Use the Intersection Point(s) tool (also from the Points & Lines menu) to locate the point where the "lower" of these lines intersects the circle (Figure 7).

- The line and circle both appear dashed when the intersection is found.

19. Use the Perpendicular tool from the Constructions menu to drop a perpendicular to either of the lines through that intersection point.

- Students should agree that the lines are parallel by the definition of a square, and thus the new line is indeed perpendicular to both lines.

20. Use the Intersection Point(s) tool again to locate the intersection of this perpendicular and the "upper" of the parallel lines (Figure 8).

21. Construct a rectangle such that this intersection point is the upper-left vertex and the right side of the square is the right side of the rectangle.

22. To ensure that the construction will remain intact when it is resized later, the rectangle should be constructed using the Polygon tool from the Shapes menu, selecting each of the vertices of the desired rectangle in order and finally double-clicking on the last vertex (Figure 9).

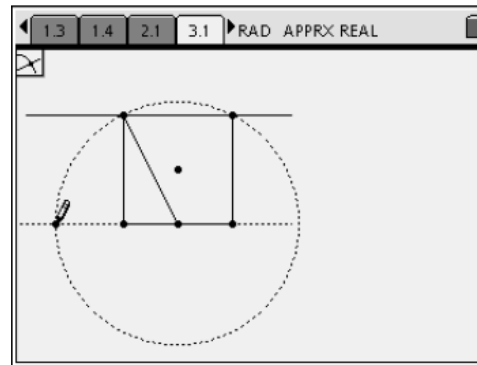**23.** Use the Length tool (from the Measurement menu) to label the dimensions of the rectangle.
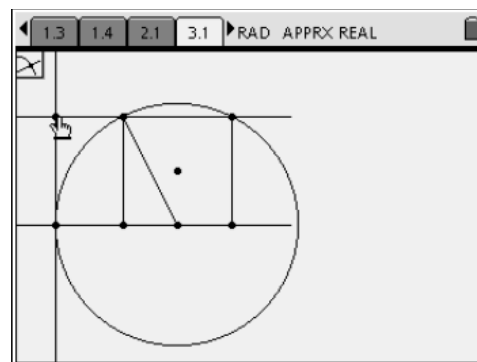

Figure 7
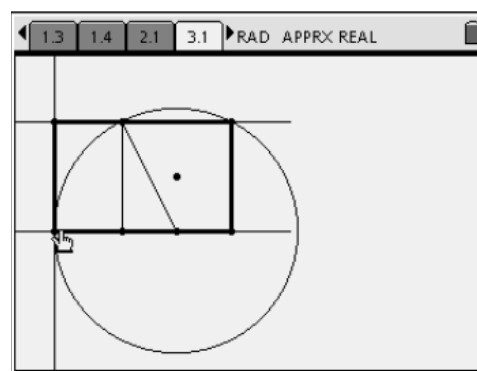

Figure 8

Figure 9

24. To use the Length tool to measure a side, select the two endpoints of that side.

   - Select the side by clicking on it and not its endpoints will give the perimeter of the rectangle.

25. Press (menu) (1) (5) for Menu 1:Actions, 5:Text to display the expression $\dfrac{l}{w}$ in an open area of the screen (Figure 10).

Figure 10

26. Calculate the ratio of length to width by pressing (menu) (1) (7) for Menu 1:Actions, 7:Calculate and clicking on the expression $\dfrac{l}{w}$ (Figure 11).

   - In moving the cursor off of the expression, you are then prompted for the value of each variable.

Figure 11

27. Click on each displayed measurement in turn, and click again in an open area to display the ratio.

   - Students should recognize this number!

28. Drag a corner of the original square and observe that as the dimensions of the rectangle change, the ratio remains the same.

29. On page 3.2, have students explore another construction involving the Golden Ratio.
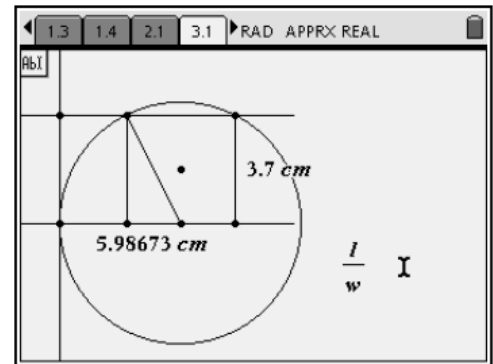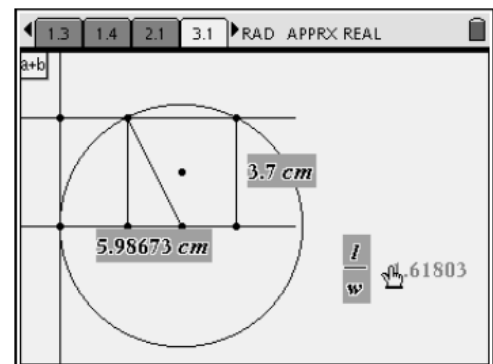
30. Construct a regular decagon (using the Regular Polygon tool from the Shapes menu) and a circle that circumscribes the decagon (Figure 12).

31. The circle is drawn by selecting the Circle tool from the Shapes menu, clicking first on the center and then on a vertex of the decagon.

32. Draw a radius (Segment tool) and calculate the ratio of the radius to the length of a side of the decagon (See Figure 12).

- Again, students will see a familiar approximation.

Figure 12

Problem 4 – What is this Number?

33. Share with students that the exact value of this number is given by the expression $\frac{1+\sqrt{5}}{2}$, and it is known as the *Golden Ratio*.

- A rectangle that has sides in this ratio, like the one the explored on page 3.1, is called a *Golden Rectangle*.

- It is said that such a rectangle is the most aesthetically pleasing of all rectangles. These rectangles can be found in both ancient and modern art and architecture.

34. Use the Calculator application on page 4.1 to find a decimal approximation for $\frac{1+\sqrt{5}}{2}$ (Figure 13).
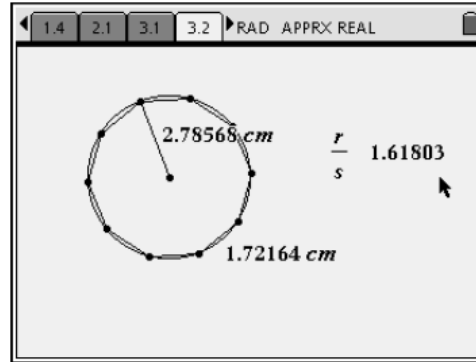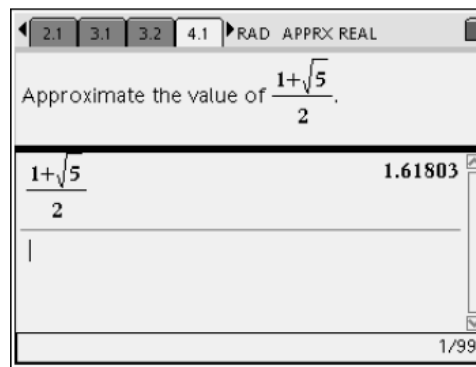

Figure 13

35. Access the division template by pressing (ctrl) (÷).

- That and more templates can be accessed by pressing (ctrl) (▦).

36. Remind students that because $\sqrt{5}$ is an irrational number, the Golden Ratio $\frac{1+\sqrt{5}}{2}$ is also an irrational number. It never ends and never repeats.

37. You may want to have students adjust the Document Settings (press (⌂)) to view more decimal places.

Problem 5 – Extension

38. Find the distance from the top of their head to their elbow (when their arms are down against their sides).

- The ratio of the distance from the top of their head to their fingertips (found in the lesson), to this distance, will also be near the Golden Ratio.

- Encourage students who find these ratios intriguing to research other ratios that form the Golden Ratio.

The Golden Ratio

(Student)TI-Nspire™ Document: *GeoAct04_GoldenRatio_EN.tns*

1.1 1.2 1.3 1.4 ▶RAD APPRX REAL

THE GOLDEN RATIO

Geometry

An interesting number with
many appearances

1.1 1.2 1.3 1.4 ▶RAD APPRX REAL

On the next page…

* Choose a 2⁻ or 3⁻digit number.
* Find its square root, and add 1.
* Then find the square root of that
  number, and add 1.
* Repeat the above several times—
  end with taking the square root.

1.1 1.2 1.3 1.4 ▶RAD APPRX REAL

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1.1 1.2 1.3 1.4 ▶RAD APPRX REAL

Try the next number pattern here.

[ ]

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1.2 1.3 1.4 2.1 ▶RAD APPRX REAL

Find the ratio described on the worksheet.
Then calculate the mean ratio for your class.

[ ]

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1.3 1.4 2.1 3.1 ▶RAD APPRX REAL

1.4 2.1 3.1 3.2 ▶RAD APPRX REAL

2.1 3.1 3.2 4.1 ▶RAD APPRX REAL

Approximate the value of $\dfrac{1+\sqrt{5}}{2}$.

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3.1 3.2 4.1 5.1 RAD APPRX REAL

Extension

Find the ratio of the distance from the top of
your head to your fingertips (found in
Problem 2) to the top of your head to your
elbow. Is it close to the Golden Ratio?
Research other measurements of humans
and animals that are related to this ratio.