

Angle Bisectors in a Triangle-Teacher

Concepts

- Relationship between an angle bisector and the arms of the angle
- Applying the Angle Bisector Theorem and its converse

Materials

- TI-Nspire™ Math and Science Learning Handheld
- *PTE-Geom_AngleBis_EN.tns*

Overview

In this activity, students will explore the relationships between an angle bisector and segments in a triangle. They will determine the distances from an angle bisector to the sides of the bisected angle. In a triangle, proportional relationships occur when an angle bisector divides the opposite side into two parts.

Teacher Preparation

This activity is designed to be used in a high school or middle school geometry classroom.

The Angle Bisector Theorem states “If a point is on the bisector of an angle, then it is equidistant from the sides of the angle.”

In a triangle, when an angle bisector divides the opposite side into two parts, the segments created will be proportional to the adjacent sides. For the diagram used in Problem 2, the following proportions will both be true:

$$\frac{BD}{AB} = \frac{CD}{AC} \quad \text{and} \quad \frac{BD}{CD} = \frac{AB}{AC}$$

Problem 3 is an optional extension involving the incenter of the triangle, which is the point of concurrency of all three angle bisectors.

The screenshots within the activity demonstrate expected student results. Refer to the screenshots at the end of this activity for a preview of the student TI-Nspire™ .tns document.

Classroom Management

This activity is designed to be student-centered with the teacher acting as a facilitator while students work cooperatively. Use the following pages as a framework as to how the activity will progress.

The student worksheet helps guide students through the activity and provides a place for students to record their answers and observations.

Problem 1- The Angle Bisector Theorem

1. Have students open the file *PTE_Geom_AngleBis_EN.tns* and read the directions on page 1.2.
2. On page 1.3, $\angle BAC$ has been constructed.
3. Students are to construct the angle bisector of $\angle BAC$ using the Angle Bisector tool (menu 9 4) for Menu 9:Construction, 4:Angle Bisector) (Figure 1).
4. Direct students to place a new point on the angle bisector.
5. Label this point X (Figure 2).
 - If the point was not labeled as it was created, select the Text tool by pressing (menu 1 5) for Menu, 1:Actions, 5:Text.

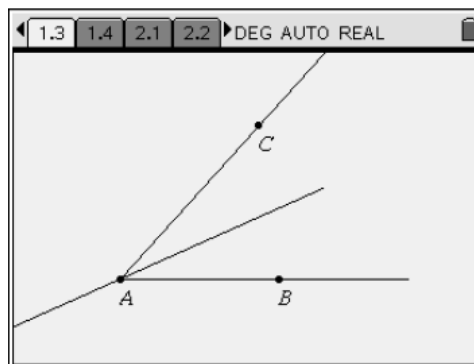


Figure 1

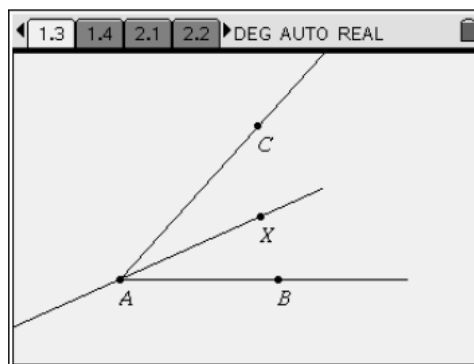


Figure 2

6. Have students measure angles $\angle BAX$ and $\angle CAS$ by pressing (menu) 7 4 for Menu 7:Measurement, 4:Angle (Figure 3).
7. Drag point B or C, and observe the results.
 - Does your observation confirm the definition of an angle bisector?
8. If you want, hide the angle measures with the Hide/Show tool by pressing (menu) 1 2 for Menu 1:Actions, 2:Hide/Show.
 - The distance from point X to the sides of the angle must be measured perpendicularly.
9. Students will construct a line through X perpendicular to AB using the Perpendicular tool ((menu) 9 1 for Menu 9:Construction, 1:Perpendicular) (Figure 4).
10. Repeat to construct a line through X perpendicular to AC (See Figure 4).
11. Students should use the Intersection Point(s) tool ((menu) 6 3 for Menu 6:Points & Lines, 3:Intersection Point(s)) to place points at the intersection of \overline{AB} and its perpendicular line and the intersection of \overline{AC} and its perpendicular line.
12. Hide the perpendicular lines (Figure 5).
13. Have students use the Segment tool ((menu) 6 5 for Menu 6: Points & Lines, 5:Segment) to connect X to each intersection point (Figure 6).
14. Measure the lengths of each segment using the Length tool ((menu) 7 1 for Menu 7:Measurement, 1:Length).

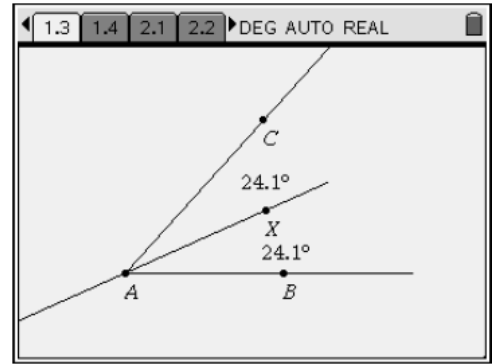


Figure 3

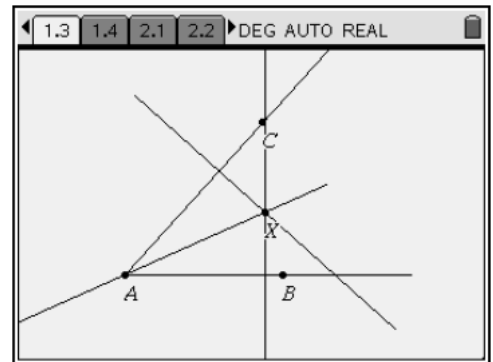


Figure 4

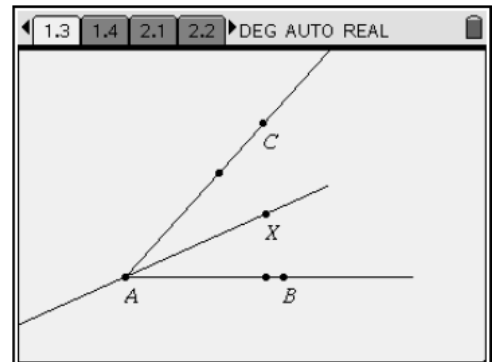


Figure 5

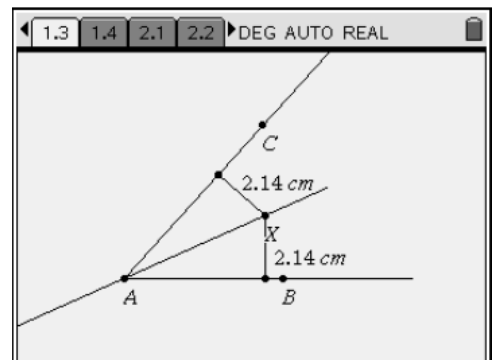


Figure 6

15. Students should drag point X and observe the changes in the measurements (Figure 7).

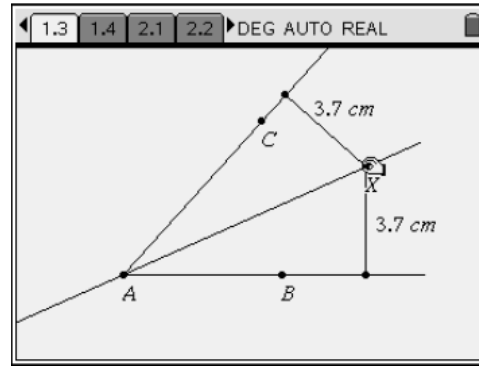


Figure 7

16. Then drag point B or C to change the size of the angle, and observe the results (Figure 8).

17. Record observations on the worksheet.

Problem 2 – One Angle Bisector in a Triangle

18. Students should advance to page 2.1 and read the directions.

19. On page 2.2, students are to construct the angle bisector of $\angle BAC$ (Figure 9).

20. Before continuing, have students drag a vertex of $\triangle ABC$ so that everyone gets different data.

21. Have students find the intersection point of the angle bisector and side \overline{BC} .

22. Label this point D (Figure 10).

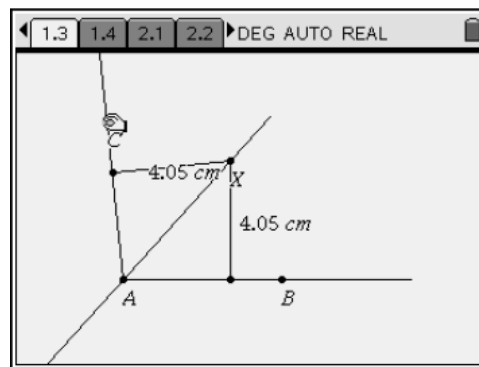


Figure 8

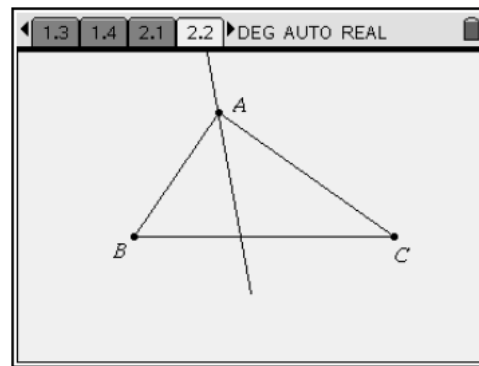


Figure 9

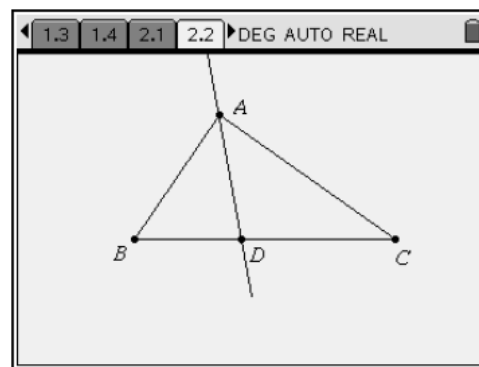


Figure 10

23. Direct students to measure the lengths of \overline{AB} , \overline{AC} , \overline{BD} , and \overline{CD} using the Length tool (Figure 11).

24. Record these values on the worksheet.

25. Drag a vertex of the triangle, and record more data.

26. Have students use the Text tool to put the expression $\frac{p}{q}$ on the screen (Figure 12).

- They will use this expression to calculate ratios of the measurements.

27. Divide pairs of the measurements using the Calculate tool (menu 1 7 for Menu 1:Actions, 7:Calculate).

28. Examine the ratios that result (Figure 13).

29. Drag a vertex of $\triangle ABC$ and examine the ratios again. What do you notice?

30. Ask students to identify a set of ratios that are equal to each other.

31. Drag a vertex of the triangle to confirm this (Figure 14).

32. Record observations on the worksheet.

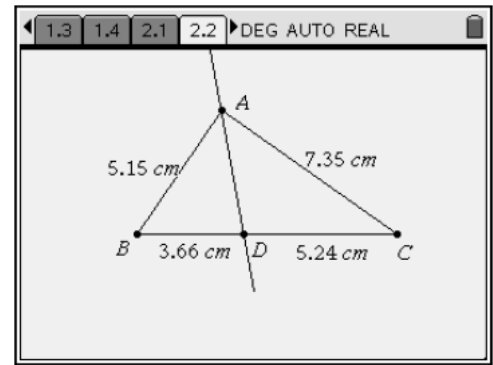


Figure 11

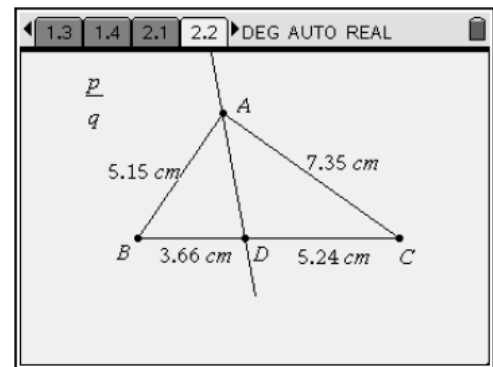


Figure 12

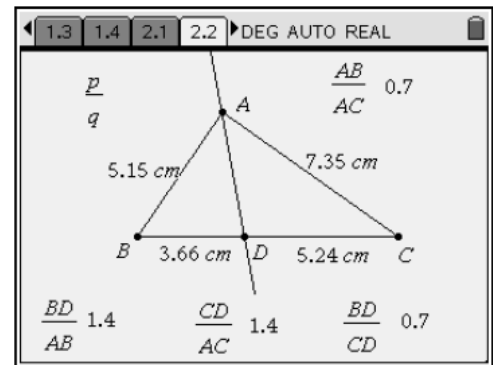


Figure 13

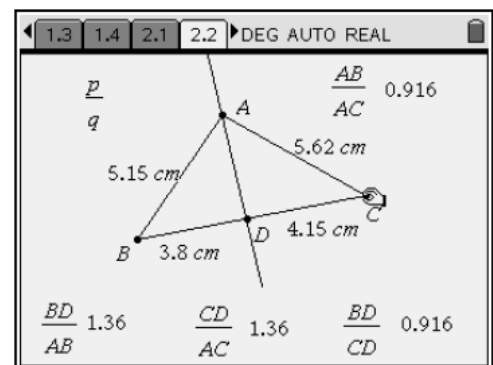


Figure 14

Extension: One Angle Bisector and the Incenter of a Triangle

33. Students should advance to page 3.1 and read the directions.

- On page 3.2, $\triangle DEF$ is constructed with all three angle bisectors created (Figure 15).
- The point at which the three angle bisectors intersect is called the incenter and is labeled I.

34. Students should hide angle bisectors \overleftrightarrow{EI} and \overleftrightarrow{FI} using the Hide/Show tool (Figure 16).

35. Use the Intersection tool to find the point of intersection of the remaining angle bisector with the opposite side and label it point G.

36. Direct students to measure the lengths of \overline{DI} and \overline{DG} with the Length tool (Figure 17).

37. Also measure sides \overline{DE} and \overline{DF} and the perimeter of the triangle.

- The expressions $\frac{DI}{DG}$ and $\frac{DE+DF}{P}$ are on the screen.

38. Students should use the Calculate tool to evaluate these expressions (Figure 18).

39. Compare the quotients to each other.

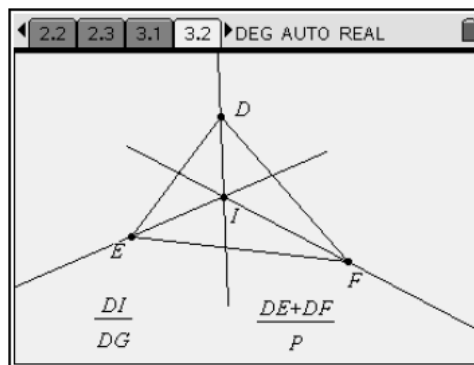


Figure 15

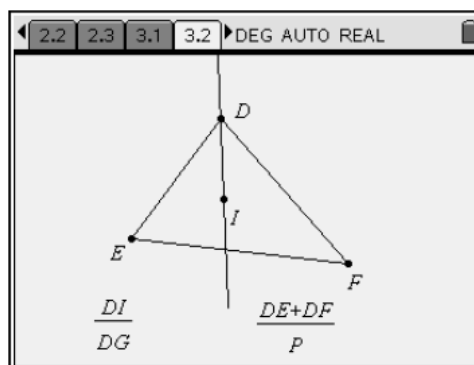


Figure 16

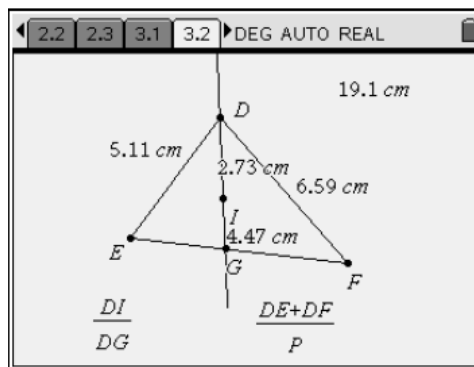


Figure 17

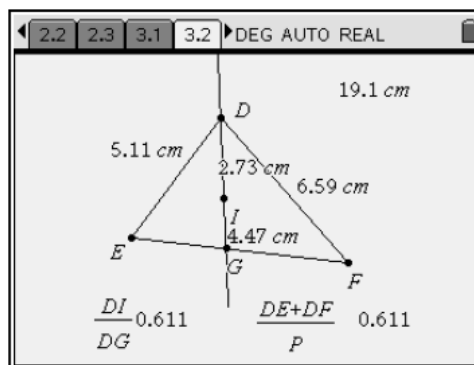


Figure 18

40. Students should drag a vertex of $\triangle DEF$ and examine the quotients again (Figure 19).

- What do you notice?

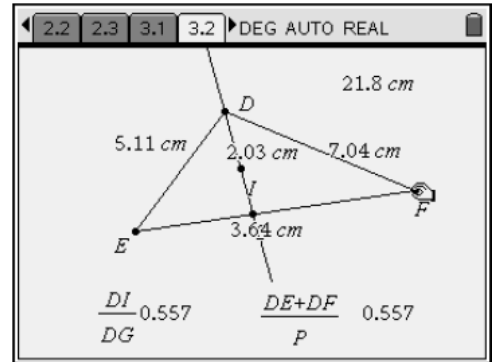


Figure 19

Angle Bisectors in a Triangle

(Student)TI-Nspire™ Document: *Geom_AngleBis_Student.tns*

1.1 1.2 1.3 1.4 ▶DEG AUTO REAL

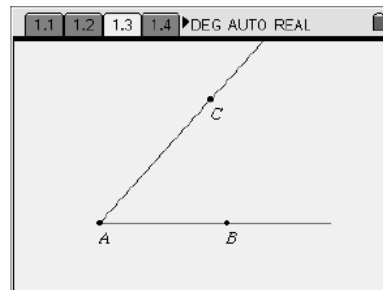
**ANGLE BISECTORS
IN A TRIANGLE**

Geometry
Angle Bisectors

1.1 1.2 1.3 1.4 ▶DEG AUTO REAL

On the next page, $\angle BAC$ is shown.

Construct the angle bisector of $\angle BAC$. Place a new point X on the angle bisector.



1.1 1.2 1.3 1.4 ▶DEG AUTO REAL

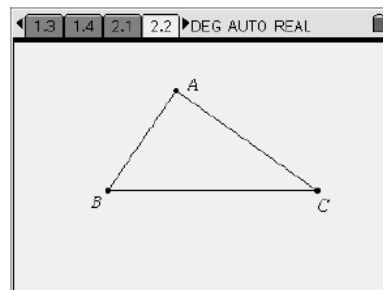
On page 1.3, measure the perpendicular distance from X to rays AB and AC .

What do you notice?

1.2 1.3 1.4 2.1 ▶DEG AUTO REAL

On the next page, $\triangle ABC$ is shown.

Construct the angle bisector of $\angle BAC$.



1.4 2.1 2.2 2.3 ▶DEG AUTO REAL

On page 2.2, measure the lengths AB , AC , BD , and CD .

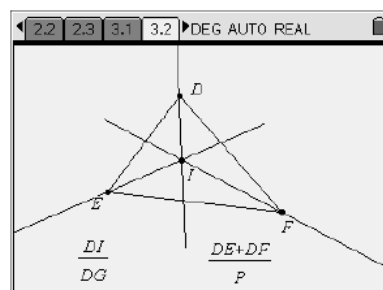
Find the ratios of pairs of these measurements. What do you notice about the results?

2.1 2.2 2.3 3.1 ▶DEG AUTO REAL

Optional Extension:

Triangle DEF appears on page 3.2. In $\triangle DEF$, three angle bisectors intersect at the *incenter* I . Hide bisectors EI and FI .

Place a point at the intersection of the angle bisector and segment EF and label it G .



2.3 3.1 3.2 3.3 ▶DEG AUTO REAL

Measure the lengths DI , DG , DE , DF and the perimeter of the triangle.

Evaluate the ratios on the screen and compare them.