Make the Basket – ID: 10221
By Pat Flynn

Time required
30 minutes

Activity Overview
Students will use parametric equations to model two physical situations: making a free throw (basketball) and hitting a home run (baseball). Students will begin exploring the models by using sliders to change to the angle and velocity of the shot or hit. They will then move the time slider to see how the model behaves with respect to time. This will reinforce the idea that the parametric equations are determining a specific location in space at a certain time, t.

Concepts
- *Parametric equations*
- *Projectile motion*

Teacher Preparation
This investigation offers an approach to show students how parametric equations can be used in real-world situations.

- *The screenshots on pages 2 and 3 demonstrate expected student results. Refer to the screenshots on page 4 for a preview of the student TI-Nspire document (.tns file).*

- ***To download the student .tns file and student worksheet, go to education.ti.com/exchange and enter "10221" in the quick search box.***

Classroom Management
- *This activity is intended to be **teacher-led**. You may use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds. The majority of the ideas and concepts are only presented in this document, so be sure to cover all the material necessary for students' total comprehension.*

- *The student worksheet PreCalcAct33_MakeTheBasket_worksheet_EN is intended to guide students through the main ideas of the activity, while providing more detailed instruction on how they are to perform specific actions using the tools of the TI-Nspire handhelds. It also serves as a place for students to record their answers. Alternatively, you may wish to have the class record their answers on separate sheets of paper, or just use the questions posed to engage a class discussion.*

TI-Nspire™ Applications
Calculator, Graphs & Geometry, Notes

Introduction – The path of a projectile

When an object is thrown forward through the air, it generally follows a parabolic path. Students are asked to sketch such a path on their worksheets, like the one shown below.

This part of the activity may be the most difficult for the students to grasp. Their worksheet guides them through the following derivation of parametric equations modeling projectile motion, but it is important that you provide additional assistance as needed.
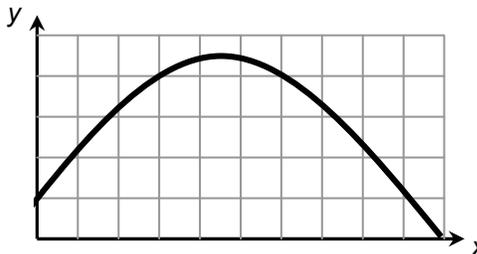
Some factors that influence the path of the object include:

- the initial launch height of the object, h_0
- the initial velocity of the object, v_0.
- the angle at which the object was launched, θ
- the effect of gravity upon the object, -16 ft/sec^2

Each point on the graph of a projectile's path represents a location at a time t. The location has both a horizontal and a vertical component: the horizontal distance, x, from its initial starting point, and the vertical distance, y, from its initial launch height.

How far the object has traveled horizontally and vertically depends on the initial velocity, elapsed time in the air, and the angle at which it was launched. The initial velocity of is a vector, where the horizontal and vertical velocities affect the horizontal and vertical distances. The initial velocity can be illustrated using the right triangle shown here.
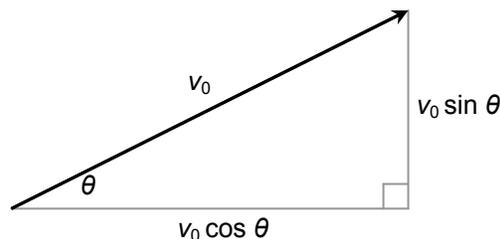
Using some trigonometry, the equations for the horizontal and vertical distances are given by the parametric equations:
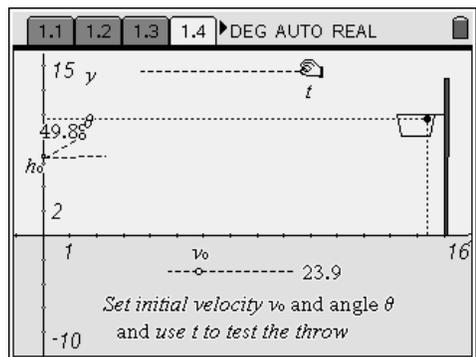
$$x(t) = v_0 \left(\cos\theta \right) t$$
$$y(t) = -16t^2 + v_0 \left(\sin\theta \right) t + h_0$$

Problem 1 – Free throws

Page 1.4 allows students to explore free throws in basketball. There are sliders for v_0, θ, and t, and the value of h_0 is fixed at 6.5 feet. Once all students in the class have determined a winning shot, it could be interesting to compare the lengths, in seconds, of their shots.

One such solution is shown to the right.

To graph the parametric equations on page 1.5, students will first need to select **MENU > Graph Type > Parametric**. Again, this is only an example of a solution—many different answers are possible.
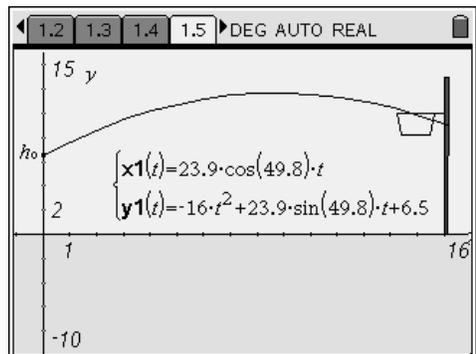
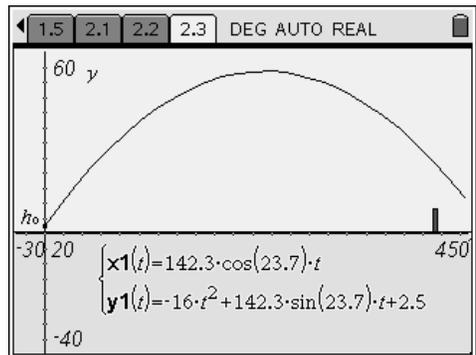**Problem 2 – A home run**

Page 2.2 allows students to explore hitting an over-the-fence home run in baseball. Again, there are sliders for v_0, θ, and t, and the value of h_0 is fixed at 2.5 feet.

One such solution is shown to the right.
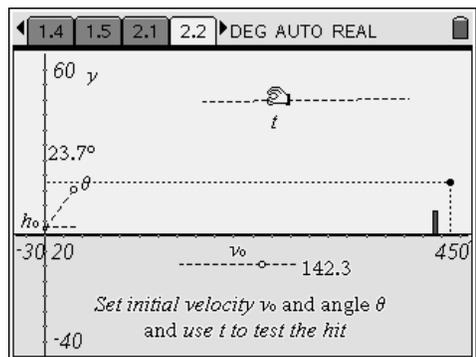
Once again, students should graph their parametric equations to view the path of the baseball and determine if it indeed went over the fence.

Make the Basket – ID: 10221
(Student)TI-Nspire File: *PreCalcAct33_MakeTheBasket_EN.tns*

| 1.1 | 1.2 | 1.3 | 1.4 | ▶DEG AUTO REAL | 🔋 |

MAKE THE BASKET

Precalculus
Parametric equations

| 1.1 | 1.2 | 1.3 | 1.4 | ▶DEG AUTO REAL | 🔋 |

Refer to your student worksheet to learn how the parametric equations below can be used to model the path of a projectile in motion.

$x(t) = v_0(\cos \theta)t$

$y(t) = -16t^2 + v_0(\sin \theta)t + h_0$

| 1.1 | 1.2 | 1.3 | 1.4 | ▶DEG AUTO REAL | 🔋 |

The construction on the next page models a person shooting a free throw in basketball. Assume h_0=6.5 and use the sliders to determine the initial velocity, v_0, and angle, θ, so that the ball will go through the basket. Then graph parametric equations on page 1.5 to model the scoring throw.

| 1.1 | 1.2 | 1.3 | 1.4 | ▶DEG AUTO REAL | 🔋 |

15 y
------o------------ t
30.7° θ
h_0
2
1   v_0 16
------o--------- 33.3
Set initial velocity v_0 and angle θ
and use t to test the throw
-10

| ◀ 1.2 | 1.3 | 1.4 | 1.5 | ▶DEG AUTO REAL | 🔋 |

15 y
h_0
2
1   16
-10

| ◀ 1.3 | 1.4 | 1.5 | 2.1 | ▶DEG AUTO REAL | 🔋 |

The construction on the next page models a person hitting a baseball. Assume h_0=2.5 and use the sliders to determine the initial velocity, v_0, and angle, θ, that results in an over-the-fence home run. Then graph parametric equations on page 2.3 to model the home run.

| ◀ 1.4 | 1.5 | 2.1 | 2.2 | ▶DEG AUTO REAL | 🔋 |

60 y
------o------------------ t
24.9°
θ
h_0
-30 20 v_0 450
------o------- 92.4
Set initial velocity v_0 and angle θ
and use t to test the hit
-40

| ◀ 1.5 | 2.1 | 2.2 | 2.3 | DEG AUTO REAL | 🔋 |

60 y
h_0
-30 20   450
-40