

Lesson Plan Notes for Derivatives for Algebra 1

NCTM Standards

All students should...

Grades 9-12 Algebra Standards:

1. Generalize patterns using explicitly defined and recursively defined functions.
2. Analyze functions of one variable by investigating: rates of change, intercepts, zeros, asymptotes, and local and global behavior.
3. Understand and perform transformations such as arithmetically combining, composing, and inverting commonly used functions, and using technology to perform such operations on more-complicated symbolic expressions.
4. Understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions.
5. Interpret representations of functions of two variables.
6. Use symbolic algebra to represent and explain mathematical relationships.
7. Judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.
8. Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships.
9. Draw reasonable conclusions about a situation being modeled.

Activity Overview

As students drag point X along the x -axis, the tangent line “slides” along the graph of the parabola and the slope of the tangent line changes. The slope of the tangent line depends

on the shape of the graph and the point of tangency. Students should be able to verbalize these relationships from their observations.

- They should then construct a locus that shows the path of point P .
- They should be able to describe how this locus summarizes information about the slope of the parabola.
- Students will then change the parabola to other functions such as a cubic or a sine wave and repeat the process.

The technique of constructing perpendicular lines through point X and the transferred point on the y -axis is called “cross-hairing” (like the cross-hairs in a telescopic sight or surveyor’s transit). In a mathematical sense, these two perpendicular lines project their individual values across the plane allowing the change of the two variables to be viewed as the pathway of this point (Figure 1).

Materials Needed

“D2_3Derivative.tns” document

Prerequisite Knowledge

Students should be familiar with the concept of the derivative as the slope of a tangent line. However, they should not know about the derivative as a function since this activity will help them discover this new way of thinking about derivatives.

Pedagogy

Students are sometimes confused in Calculus when the term *derivative* is used in one context to represent a numeric value and used in another context to represent a function. This activity should help them understand these two uses of the derivative.

Properties that may need some prompting from the teacher are related to special positions of the tangent line, e.g., when its slope is zero, when the direction of rotation changes, why the graph of the derivative looks the way it does, or what happens off the edges of the screen.

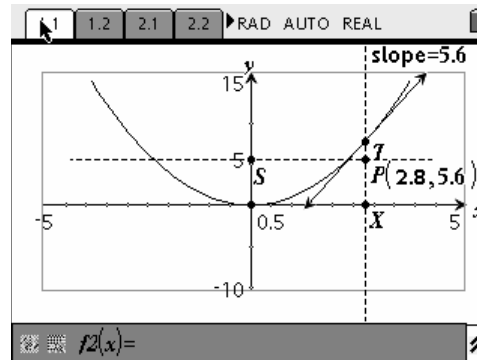



























Figure 1

The idea of mapping the value of the slope of the tangent line to the y -axis is the key element in the visual technique of this activity. Once this value is mapped to the y -axis, the stage is set to produce the graph of a function “derived” from a variable dependent on the actual shape of the original function’s graph.

Instructions to Create the Document

- To open a new document, press   for Home 6:New Document.
- Press  for 2:Add Graphs & Geometry application.
 - Enter the function $f1(x) = x^2$ in the function edit line (Figure 2).
 - Press    for Menu 4:Window, 1:Window Settings, to set a $[-5,5]$ x $[-10,15]$ viewing window.
 - Press    for Menu 1:Tools, 2:Hide/Show, to hide the equation $f1$ in the viewing window.
- Press    for Menu 6:Points & Lines 2:Point On to place a point on the x -axis.
 - Place the point between hash marks.
 - After placing the point on the axis, press   to attach a label to the point.
- Press    for Menu 9:Construction, 1:Perpendicular.
 - Place a line perpendicular to the x -axis at point X .
- Press    for Menu 6:Points & Lines, 3:Intersection Point(s).
 - Mark the intersection of the perpendicular line and the graph of $f1(x) = x^2$.
 - Press   to label the point “ T ” (Figure 3).
- Press    for Menu 6:Points & Lines, 7:Tangent to create a tangent line to the curve at T .
 - Be certain the point T is blinking (selected) when you place the tangent line.

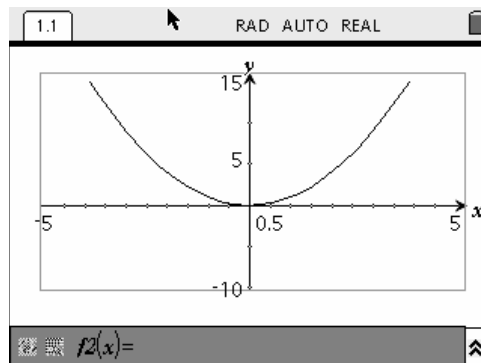


Figure 2

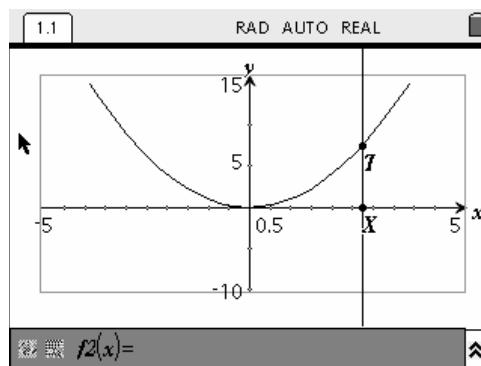


Figure 3

7. Press **(menu)** **7** **3** for Menu 7:Measurement, 3:Slope to find the slope of the tangent line.
 - After clicking the tangent line, the value of the slope should appear.
 - Move this value to the upper right-hand corner of the screen (Figure 4).
8. Press **(menu)** **1** **5** for Menu 1:Tools, 5:Text to insert the label “slope =” in front of the slope value.
 - Drag the label to adjust its placement.
 - Adjust the precision of the slope to one decimal place by hovering the pointer tool over the value of the slope and pressing **(◂)** several times (Figure 5).
9. Press **(menu)** **9** **8** for Menu 9:Construction, 8:Measurement to transfer the value of the slope on to the y-axis.
 - A point should appear on the y-axis.
 - Label this point S.
 - Press **(menu)** **9** **1** for Menu 9:Construction, 1:Perpendicular to construct a perpendicular line to the y-axis through point S.
 - Press **(menu)** **1** **3** for Menu 1:Tools, 3:Attributes to change the style of the two perpendicular lines to *dashed* (Figure 6).
10. Find the intersection of the perpendicular lines by pressing **(menu)** **6** **3** for Menu 6:Points & Lines, 3:Intersection Point(s).
 - Label this point *P*.
 - Press **(menu)** **1** **6** for Menu 1:Tools, 6:Coordinates and Equations to find the coordinates of point *P*.
 - Adjust the precision of the coordinates to one decimal place by hovering the pointer tool over each coordinate and pressing **(◂)**.
 - Move the coordinates, but close to point *P*.
11. Drag point *X* along the x-axis.
 - The tangent line, the slope, and point *P* should adjust accordingly (Figure 7).

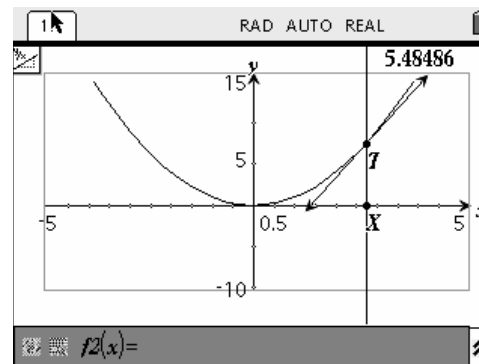


Figure 4

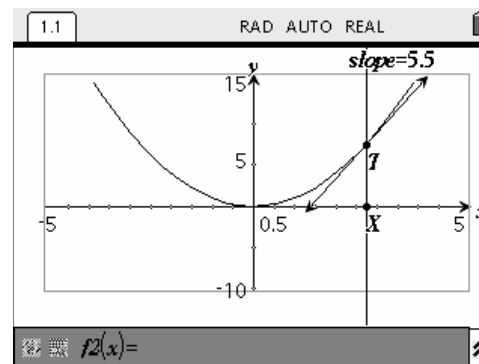


Figure 5

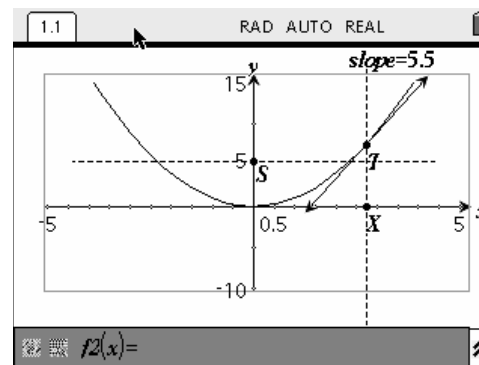


Figure 6

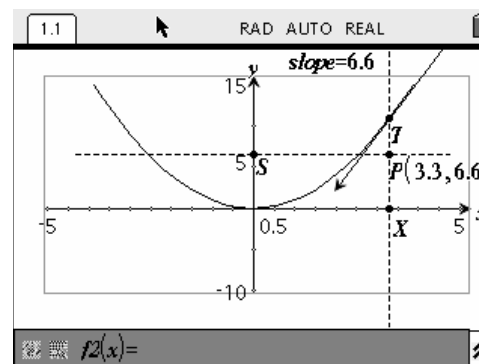
























Figure 7

12. Store the x -coordinate of point P to a variable named px .
 - To accomplish this task, select the x -coordinate with the pointer tool.
 - Press , and select 1:Store Var.
 - Then press    .
 - The value of the x -coordinate should now appear in bold print.
 - Press  to return to the pointer tool.
13. Store the value of the slope in a variable named $slope$.
 - To do this, select the slope value with the Pointer tool.
 - Press   for 1:Store Var.
 - Since the name “slope” already appears in the text box, just press .
 - Press  to return to the pointer tool.
14. Press   for Home 3:Lists & Spreadsheet to add a new application page.
15. Name the first column “xc” and the second column “slp” (Figure 8).
16. Link column A to the variable px for manual data capture with the following procedure:
 - Move the cursor to the formula entry cell for column A, press   for Menu 3:Data.
 - Press   for 2:Data Capture 2:Manual Data Capture.
 - A formula appears in the formula entry cell.
 - Press ,  for 3:Link To: .
 - Highlight variable px .
 - Press  to paste this variable back to the formula.
 - Press  again to save the formula (Figure 9).
17. Use a similar process to link column B to the variable $slope$ (Figure 10).
18. Press   to return to the Graph & Geometry application page 1.1.

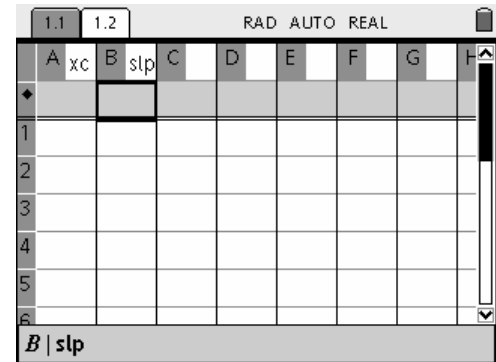


Figure 8

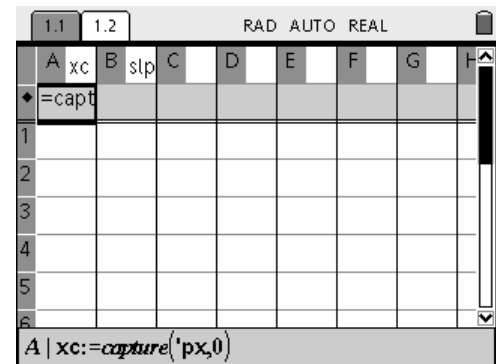


Figure 9

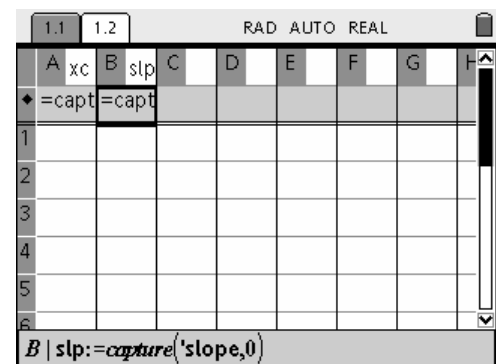


Figure 10

19. Press menu $\{3\}$ $\{3\}$ for Menu 3:Graph, Type 3:Scatter Plot.
 - Select $x\leftarrow$ for the x -values and $y\leftarrow$ for the y -values (Figure 11).
20. Press menu $\{3\}$ $\{1\}$ for Menu 3:Graph, Type 1:Function to return to function graphing mode.
21. Even though you are not finished, save the file as “tanslope.”
 - To save: Press ctrl $\{F\}$ $\{1\}$ $\{4\}$ for 1:File, 4:Save As.
 - Type in **tanslope** for the file name, and press enter .
22. Next, you are going to add another problem, and repeat steps 2 through 19.
 - However, in this problem, you will graph the cubic function $y = x^3 - 5x + 2$ in a $[-5,5] \times [-10,10]$ viewing window.
 - Press ctrl $\{F\}$ $\{2\}$ $\{8\}$ for 2:Edit, 8:Insert Problem (Figure 12).
23. Now repeat steps 2-19 with the cubic function $y = x^3 - 5x + 2$ in place of the parabola $y = x^2$.
 - Use a $[-5,5] \times [-10,10]$ window instead of a $[-5,5] \times [-10,15]$ window (Figure 13).
24. Save the updated activity by pressing ctrl $\{F\}$ $\{1\}$ $\{3\}$ for 1:File, 3:Save.
25. Now you will add one more problem, and repeat steps 2 through 19.
 - However in this problem you will add the sine function $y = 5\sin(x)$ in a $[-7,7] \times [-10,10]$ viewing window.
 - Press ctrl $\{F\}$ $\{2\}$ $\{8\}$ for 2:Edit, 8:Insert Problem.
 - Repeat steps 2 through 19 for the sine function (Figure 14).
26. Save the updated activity by pressing ctrl $\{F\}$ $\{1\}$ $\{3\}$ for 1:File, 3:Save.

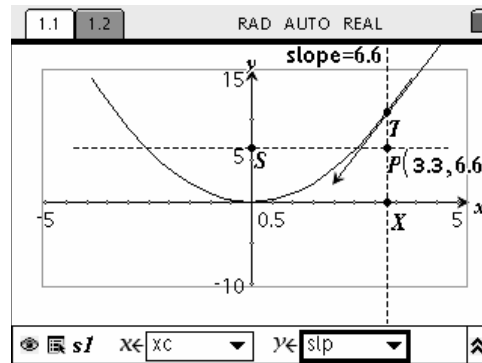


Figure 11

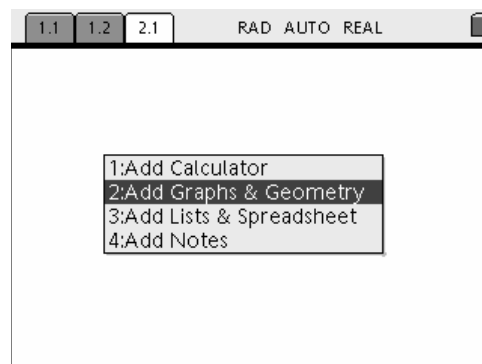


Figure 12

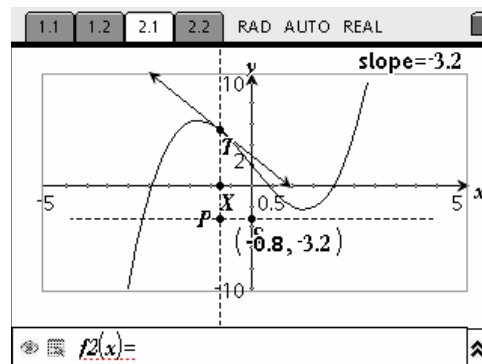


Figure 13

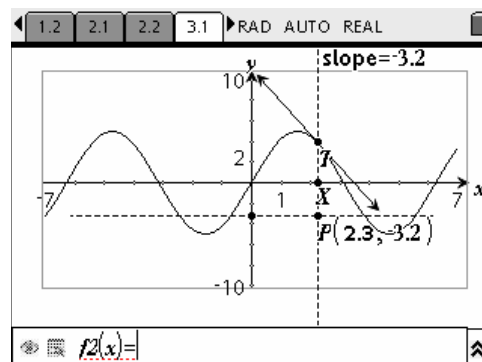


Figure 14