Advanced Algebra Nomograph

ID: 8267

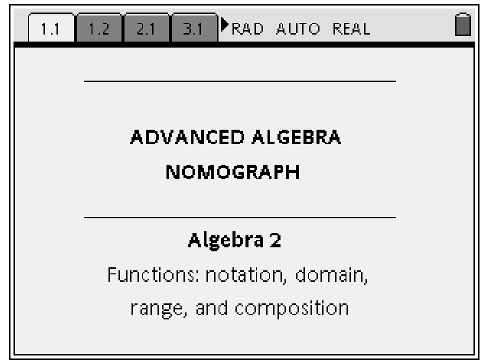
Name ______________________

Class ______________________

In this activity, you will explore:

- *functions: notation, domain, and range*
- *composite functions*

Listen as your teacher explains how the model of the nomograph works. Then open the file *Alg2Act1_AdvAlgNomograph_EN.tns* on your handheld and work with a partner to complete the activity.

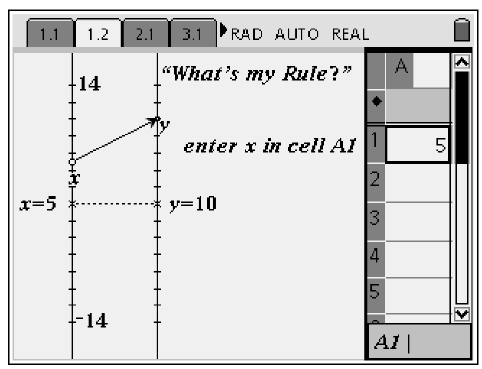


Introduction

A **nomograph** is similar to a function machine in that it relates a number in one set (the *domain*) to a number in a second set (the *range*). The nomograph takes the form of a pair of vertical number lines; the one on the left represents the domain; the one on the right represents the range. The function rule mapping an element in the domain to its corresponding element in the range is shown by an arrow.

Problem 1 – "What's my Rule?"

The first nomograph (representing an unknown function) is shown on page 1.2. Enter a value of *x* into cell A1 of the spreadsheet. (Press ctrl + tab to toggle between the applications as needed.) The nomograph relates it to a *y*-value by substituting the value *x* into the function's rule.

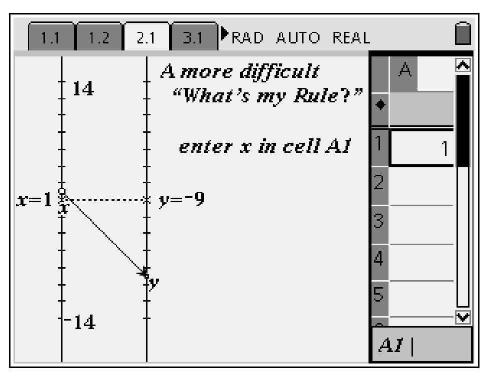


Your task is to find the "mystery rule" for **f1** that pairs each value for *x* with a value for *y*. Once you think you have found the rule, record it below. Then continue testing your prediction using the nomograph.

f1(*x*) = ___________

Problem 2 – A more difficult "What's my Rule?"

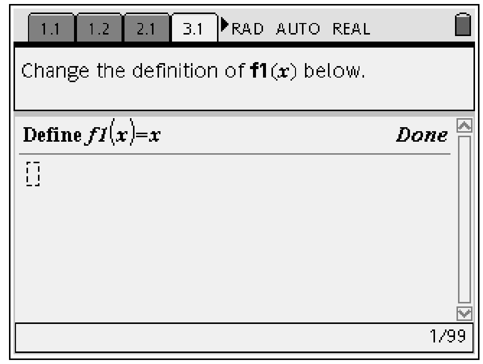
Unlike the nomograph in Problem 1, the nomograph on page 2.1 follows a non-linear function rule. As before, enter values for *x* in cell A1 and find the rule for this new function **f1**. Test your rule using the nomograph.



f1(*x*) = ___________

Problem 3 – The "What's my Rule?" Challenge

Page 3.2 shows a nomograph for the function **f1**(x) = x. The challenge is to make up a new rule (of the form $ax + b$ or $ax^2 + b$) for **f1**(x), and have a partner guess your rule by using the nomograph.

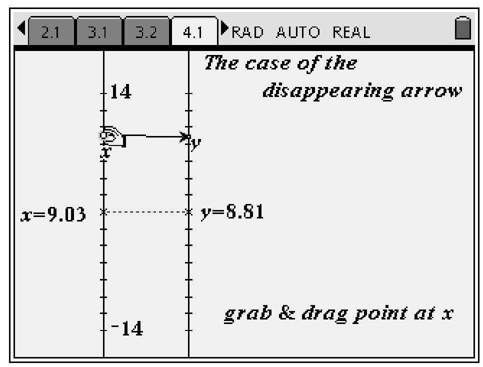


On the *Calculator* application on page 3.1, select **MENU > Tools > Recall Function Definition** and press enter to choose **f1**. Use the **CLEAR** key to erase the current definition and enter your own. Then, exchange handhelds with your partner, who will use the nomograph to discover your rule. Then, repeat.

List at least four of the functions you and your partner explored with the nomograph.

$f(x)$ = ____________ $f(x)$ = ____________ $f(x)$ = ____________ $f(x)$ = ____________

Problem 4 – The case of the disappearing arrow



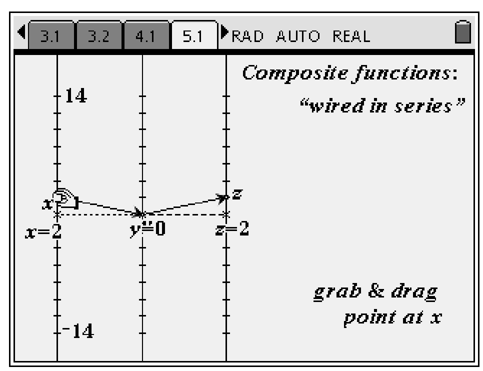
Page 4.1 shows a nomograph for the function **f1**$(x) = \sqrt{x^2 - 4}$. The input for this nomograph is changed by grabbing and dragging the base of the arrow—the point that represents x. Observe what happens when you drag this point.

When does the arrow disappear? __________________

Why does the arrow disappear? ___________________

Problem 5 – Composite functions: "wired in series"

The nomograph on page 5.1 consists of three vertical number lines and behaves like *two* function machines wired in series. The point at x identifies a domain value on the first number line and is dynamically linked by the function **f1**(x) = 3x – 6 to a range value y on the middle number line. That value is then linked by a second function **f2**(x) = –2x + 2 to a value z on the far right number line.



Either of the two notations **f2(f1(*x*))** or **f2 ◦ f1** can be used to describe the ***composite function*** that gives the result of applying function **f1** *first*, and then applying function **f2** to that result.

For example, the number 4 is linked to 6 by **f1** (because **f1**(4) = 6), which in turn is linked to –10 by **f2** (because **f2**(6) = –10). Grab and drag the base of the arrow at point x—the point "jumps" in discrete steps of 2. Set x = 4 and confirm that y = 6 and z = –10.

Find a rule for the single function **f3** that gives the same result as **f2**(**f1**(x)) for all values of x. To test your answer, move to page 5.2 and define **f3** to be your function (as you did in Problem 3). Now compute several values, for each function, such as **f2**(**f1**(4)) and **f3**(4). Are they equal?

f3(x) = ________________

Now use the *Calculator* application to compute and compare the following.

f2(**f1**(3)) = __________ **f1**(**f2**(3)) = __________

Try other values of x. Does the order in which you apply the functions matter?

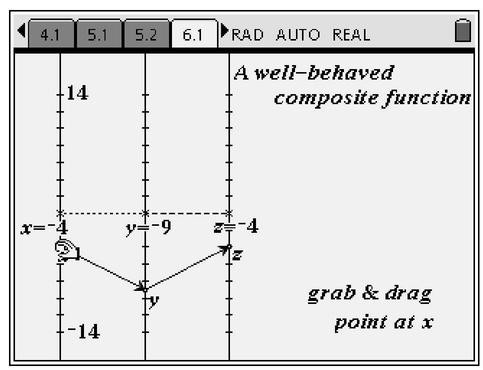
Test your understanding by completing another example:

Again on page 5.2, redefine **f1**(x) = $(x - 1)^2$ and **f2**(x) = $2x + 3$. Find a rule for both **f2** ◦ **f1** and **f1** ◦ **f2**, and test your answers by computing values as you did above. Test your answer by computing several values for each function, using the *Calculator* application.

f2(**f1**(x))= ________________ **f1**(**f2**(x))= ________________

Problem 6 – A well-behaved composite function

Some composite functions are more predictable than others. The nomograph on page 6.1 shows the function **f1**(x) = $3x + 3$ composed with a mystery function **f2**. Grab and drag the base of the arrow at x.



What do you notice about the composite function **f2** ◦ **f1**?

Play "What's my Rule?" to find the rule for **f2**.

f2(x) = ________

Now use the *Calculator* application on page 6.2 to compute and compare the following.

f2(**f1**(3)) = __________ **f1**(**f2**(3)) = __________

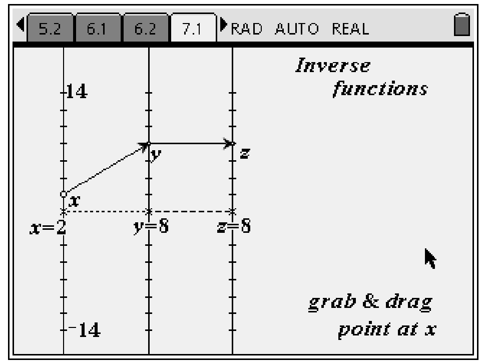
Try other values of x. Does the order in which you apply the functions matter?

Problem 7 – Inverse functions

The "inverse" of a function *f*, denoted f^{-1}, "undoes" the function—it maps a point *y* from the range back to its original *x* from the domain. You can think of a function and its inverse as a special case of function composition. (This is what was shown in Problem 6.)

By definition, **f2** is the inverse of **f1**, if and only if:

- **f2**(**f1**(*x*)) = *x* for every *x* in the domain of **f1**, and
- **f1**(**f2**(*x*)) = *x* for every *x* in the domain of **f2**.



In the context of the nomograph, **f2** is the inverse of **f1** if **f2**(**f1**(*x*)) horizontally aligns with *x* for all values in the domain of **f1** (i.e. *z* = *x*), and vice versa.

The nomograph on page 7.1 shows the composite function **f2** ◦ **f1**, where **f1**(*x*) = 2*x* + 4 and **f2**(*x*) = *x*. See if you can figure out what the rule for **f2** must be in order for **f1** and **f2** to be inverse functions. Use the *Calculator* application on page 7.2 to redefine **f2** to your rule, and return to the nomograph to test your answer.

f2(*x*) = ________________

Problem 8 – More disappearing arrows

The nomograph on page 8.1 shows the composite function **f2** ◦ **f1** where **f1**$(x) = 2x - 6$ and **f2**$(x) = \sqrt{x}$. Grab and drag the point at *x*. Watch as one of the arrows disappears.

Which arrow disappears? _____________

When and why does it disappear? ________________________________

Problem 9 – "Almost" inverses and more missing arrows

The nomograph on page 9.1 shows the composite function **f2** ◦ **f1** where **f1**$(x) = \sqrt{x}$ and **f2**$(x) = x^2$. Grab and drag the point at *x*.

When does **f2** act like the inverse of **f1**? _____________

When does **f2** *NOT* act like the inverse of **f1**? _____________

When and which arrow(s) disappears? _____________

Proceed to page 9.2 and reverse the definitions, that is, define **f1**$(x) = x^2$ and **f2**$(x) = \sqrt{x}$. Return to the nomograph.

When does **f2** act like the inverse of **f1**? _____________

When does **f2** *NOT* act like the inverse of **f1**? _____________

When and which arrow(s) disappears? _____________