Algebra Nomograph-Teacher

Concepts

- Functions, including notation, domain, and range
- Inverse functions

Materials

- TI-Nspire™ Math and Science Learning Handheld
- *PTE-Alg-AlgebraNomograph_EN.tns*

Overview

This activity is similar to the idea of a function machine. There are two levels of the manipulative(called a *nomograph*). The first is comprised of two vertical number lines, input on the left and output on the right. The second has three number lines to accommodate displaying the inverse of a function. At the first level, students try to find the rule of a hidden function by entering domain values and observing how they are transformed to new (range) values. The transformation is illustrated dynamically by an arrow that connects a domain entry to its range value. At the second level, students try to find inverses of given functions. An inverse is illustrated by a second arrow that connects a range value to a point on the third number line that equals (lines up with) the original domain entry.

Teacher Preparation

This activity is appropriate for students in Algebra 1 or Pre-Algebra. Problems 7 and 8, which examine inverse functions, are written to the Algebra 1 level.

Prerequisites are an introduction to functions (including the terms domain and range), function notation ("y=" and "f(x)="), and some experience graphing linear functions using slope and *y*-intercept.

Students will work on the .tns document *PTE-Alg-AlgebraNomograph_EN.tns* so it is important that the model be demonstrated to students prior to them exploring the .tns document on their own. A suggestion is to work through Problem 1 as a class.

- The screenshots within the activity demonstrate expected student results.
- Refer to the screenshots on the final pages for a preview of the student .tns document.

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Classroom Management

This activity is designed to have students explore individually and in pairs. However, an alternate approach would be to use the activity in a whole-class format. By using the computer software and the questions found on the student worksheet, you can lead an interactive class discussion on functions and their inverses.

User-input nomographs are implemented on a split-screen: Graphs & Geometry on the left and Lists & Spreadsheet on the right. Calculations that drive the implementations are hidden in the spreadsheet; the "split" is designed to expose only Column A. Caution students to leave the rest of the spreadsheet alone. Additionally, all inputs into cell A1 need to be in decimal form.

In the Graphs & Geometry work areas, instruct students that they are **not** to unhide the Entry Line.

Optional extension questions are provided at the end of this activity. These questions can be used to provide for individual differences. A .tns document for an extension activity can be created simply by copying the appropriate nomograph implementation from the document *NomographTemplates.tns*.

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A *nomograph* is similar to a function machine in that it relates a number from one set (the domain) to a number in a second set (the range). Each set of numbers is represented in a pair of vertical number lines; the domain is on the left, and the range is on the right. According to the function rule, an element of the domain is mapped to its corresponding range element, and this mapping is depicted by an arrow.

Prior to beginning Problem 1, review domain and range, and ensure that students understand how to use the model.

Problem 1 – "What's my Rule?"

The first several problems are "What's my Rule?" activities. Input values are entered, one at a time, into cell A1 of the spreadsheet. The nomograph displays the input and its corresponding output, and both values are recorded in a separate spreadsheet. By repeatedly entering different inputs, the student should be able to discover the function's rule (Figure 1).

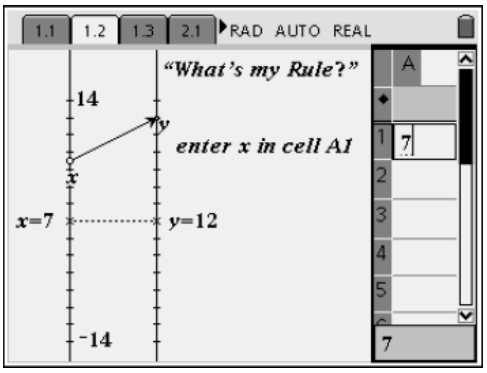

Figure 1

For example, in working with this nomograph of Problem 1, entering domain values 1, 2, 5, and 7 one at a time would yield the respective range values 6, 7, 10, and 12, and the rule $f(x) = x + 5$ should be found.

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Solutions

- Sample ordered pairs: (1, 6), (2, 7), (5, 10) (7, 12)
- **f1**(x) = x + 5
- Sample scatter plot in Figure 2.
- Possible answer: The arrows are parallel.

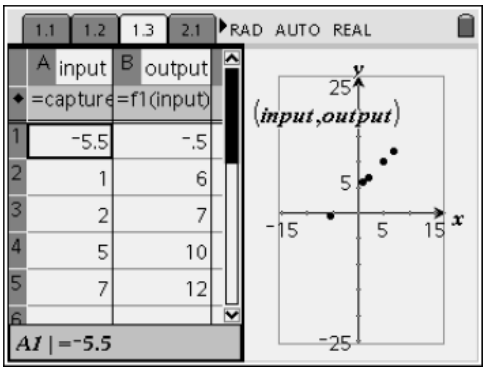

Figure 2

Problem 2 – More "What's my Rule?"

This nomograph follows a different rule than the nomograph from Problem 1 (Figure 3). Students are guided through the same steps to determine the rule.

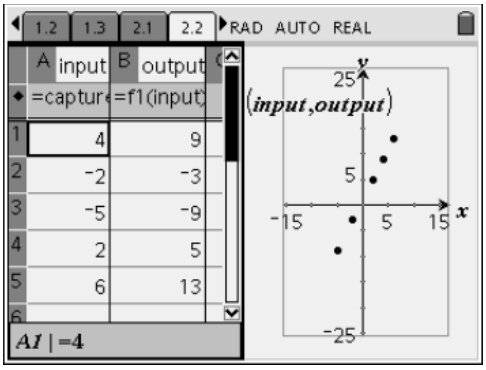

Figure 3

Solutions

- **f1**(x) = 2x + 1
- Slope: 2
- Possible answer: The arrows are spread out and are not parallel.

Problem 3 – The "What's my Rule?" Challenge

1. Instruct students to create their own functions of the form $y = ax + b$ (where a and b are integers).
2. Each student should use the Calculator work area on page 3.2 and redefine **f1** to their own function by using the Recall Function Definition command (Figure 4).

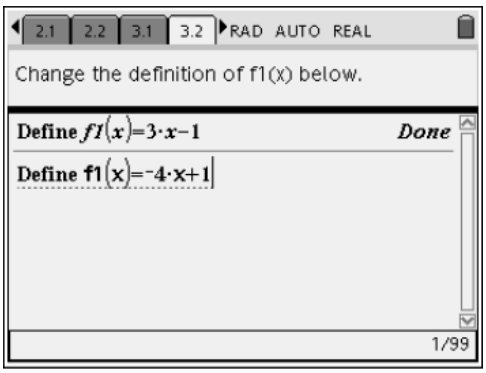

Figure 4

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3. Students should then return to page 3.1 (to display the nomograph) and exchange handhelds with a partner.
4. It is the partner's task to use the nomograph to identify the mystery function.
5. Encourage students to repeat this activity several times.

Solutions

- Functions will vary.

Problem 4 – Even more "What's my Rule?"

The next two problems are additional examples of "What's my Rule?" activities, except that the input is now controlled by dragging the point on the domain, rather than entering a value into a spreadsheet (Figure 5).

Students will need to record the ordered pairs obtained by dragging the point in order to determine the rule for the function.

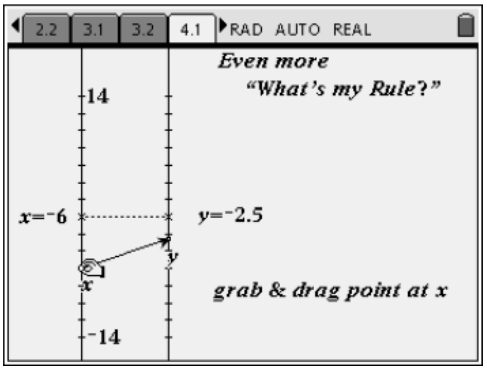

Figure 5

Solutions

- Sample ordered pairs: (12, 6.5), (10, 5.5), (8, 4.5), (4, 2.5), (–2, –0.5), (–6, –2.5)
- $y = \frac{1}{2}(x+1)$ or $y = \frac{x+1}{2}$

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Problem 5 – A more difficult "What's my Rule?"

Solutions

- Sample ordered pairs: (6, 20), (4,0), (2, –12), (0, –16), (–2, –12), (4, 0) (See Figure 6).
- $y = x_2 - 16$

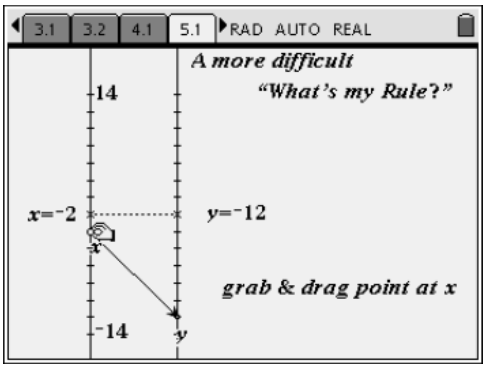

Figure 6

Problem 6 – The case of the disappearing arrow

The nomograph on page 6.1 displays a function with restricted domain: $f1(x) = \sqrt{6 - x}$. As students drag the point through *x*-values that are not in the domain, the function arrow disappears (Figure 7). Students are asked to explain when and why this happens for this specific function. To avoid confusion, make sure the arrow is visible when students first open the file (that is, $x \leq 6$).

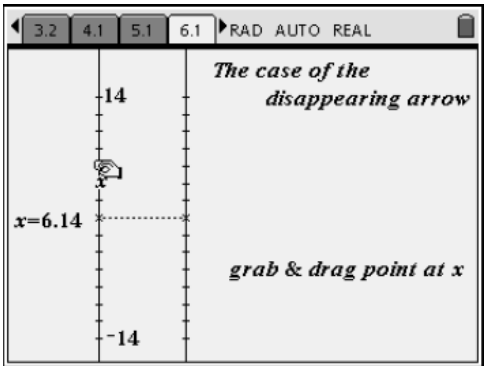

Figure 7

Solutions

- $x > 6$
- Possible answer: The square root of a negative number is undefined.

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Problem 7 – An inverse function

The next two problems have students explore the concept of an inverse function. A double nomograph is shown; the first arrow representing **f1**(x) and the second representing its inverse, **f1**$_{-1}$(x) (Figure 8). Students are encouraged to notice that the inverse function "undoes" the function; that is, **f1**$_{-1}$(**f1**(x)) = x.

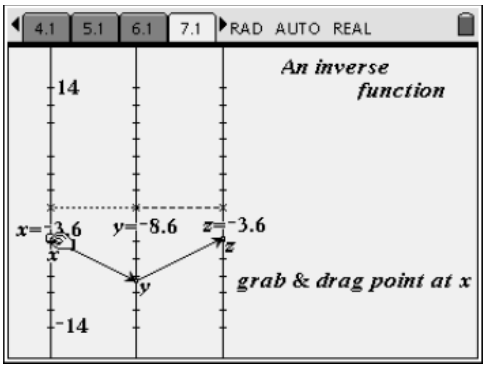

Figure 8

Solutions

- Sample ordered pairs:

  f1: (7.54, 2.54), (5.49, 0.49), (–3.6, –8.6);

  f1$_{-1}$: (2.54, 7.54), (0.49, 5.49), (–8.6, –3.6)

- Possible answer: The x- and y-coordinates in each pair are reversed.

- **f1**(x) = x – 5; **f1**$_{-1}$(x) = x + 5

Problem 8 – More inverse functions

In this final problem, students explore the inverse of a function of the form $y = ax + b$. They are also asked to look at the graphs of the function, its inverse, and the line $y = x$ and describe the relationship.

6. To unhide the graphs, press ctrl + G to unhide the Entry Line.

7. Press ▲ to access each function, and press ◀ to select each functions hide/show icon and press enter (Figure 9).

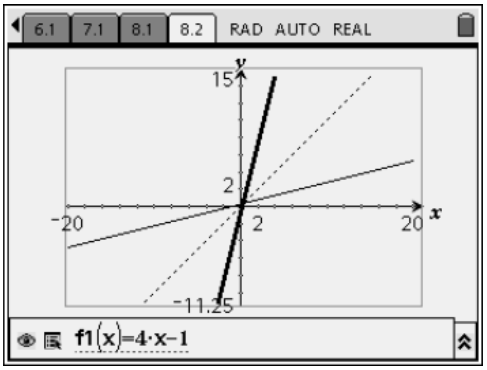

Figure 9

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Solutions

- Sample ordered pairs:

  f1: (3.09, 11.34), (0.51, 1.06), (–1.03, –5.11);

  f1$_{-1}$: (11.34, 3.09), (1.06, 0.51), (–5.11, –1.03)

  $\mathbf{f1}(x) = 4x - 1$; $\mathbf{f1}^{-1}(x) = \frac{1}{4}(x+1)$

- Possible answer: **f1**$_{-1}(x)$ is the reflection of **f1**(x) over the line $y = x$.

Algebra Nomograph – Extensions

8. Consider the following conjecture:

If f_{-1} is the inverse of f, then f is the inverse of f_{-1}.

- Is the conjecture true or false? Justify your answer.
- Note: the notation "f-1" is often used to signify the inverse of function f. It does ***not*** mean $\frac{1}{f}$.

9. Write down a function, and identify its inverse on a separate piece of paper.

- Exchange your functions with a partner, whose task is to find the inverse function.
- You can then test by entering the functions into the nomograph.
- Repeat with several different functions.

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10. The rule for **f1** that maps a number x in the domain to a number y in the range is $\mathbf{f1}(x) = x^2 + 1$.

- Work with a partner to find $\mathbf{f1}^{-1}$.
- Enter the functions $\mathbf{f1}(x) = x^2 + 1$ and $\mathbf{f2}(x) = \mathbf{f1}^{-1}$ into a double nomograph.
- Grab and drag the base of the arrow at x.
- Notice how the inverse function arrow from y to z sometimes disappears. Explain why this behavior occurs.

11. For the inverse of a function to exist, the "inverse" must map back to the original domain value for *all* values in the domain.

- Explain how you could restrict the domain for $\mathbf{f1}(x) = x^2 + 1$ so it will have a true inverse (i.e. the inverse arrow would never disappear).
- Identify at least two other functions that require restricted domains in order to have inverses.
- Restrict the domains as needed, and determine the inverse for each.

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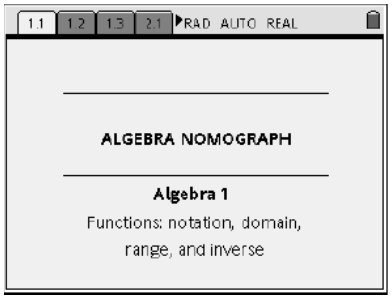
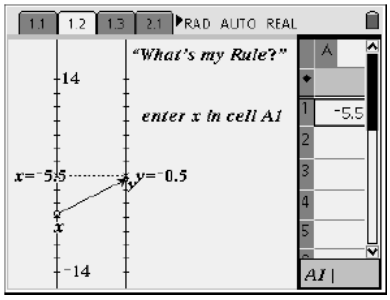
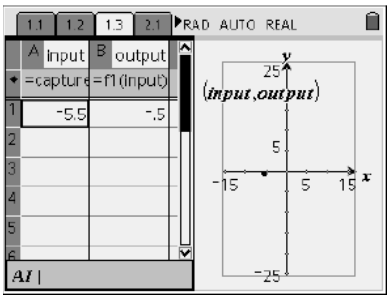
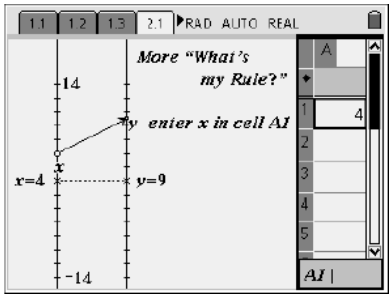
Algebra Nomograph

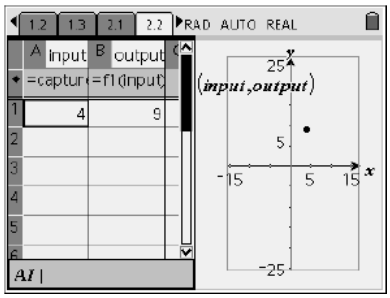
(Student)TI-Nspire™ Document: *Alg1Act2_AlgebraNomograph_EN.tns*

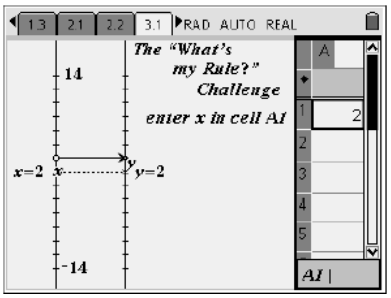


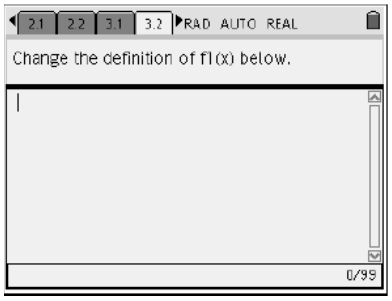


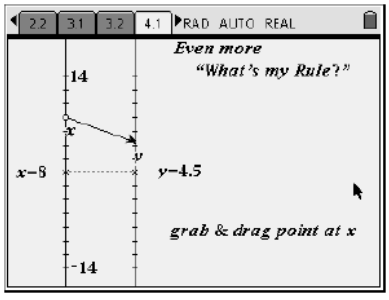


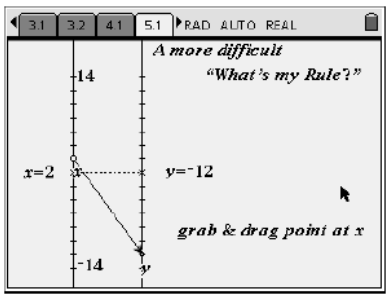


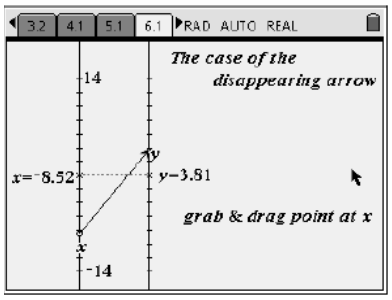


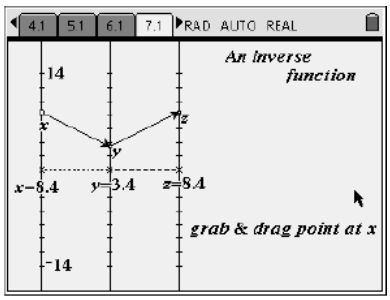


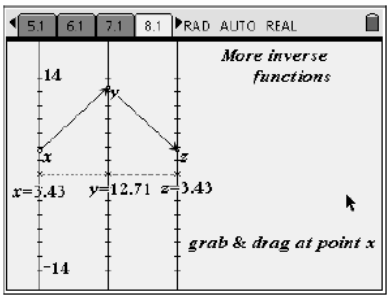












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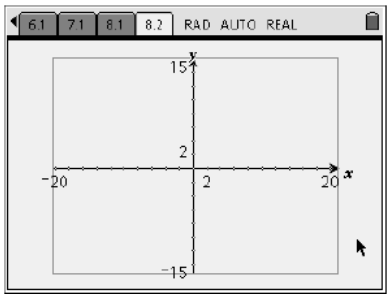

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